Vehicle Currency*

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Abstract

Historically, the world economy has been dominated by a single currency accepted in the exchange of goods and assets among countries. In recent decades, the US dollar has played this role. The dollar acts as a 'vehicle currency' in the sense that agents in non-dollar economies will generally engage in currency trade indirectly using the US dollar rather than using direct bilateral trade among their own currencies. A vehicle currency is desirable when there are transactions costs of exchange. This paper constructs a dynamic general equilibrium model of a vehicle currency. We explore the nature of the efficiency gains arising from a vehicle currency, and show how it depends on the total number of currencies in existence, the size of the vehicle currency economy, and the monetary policy followed by the vehicle currency's government. We find that there can be significant welfare gains to a vehicle currency in a system of many independent currencies. But these gains are asymmetrically weighted towards the residents of the vehicle currency country. The survival of a vehicle currency places natural limits on the monetary policy of the vehicle currency country.

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1. Introduction

The international monetary systems has usually had a predominant currency in facilitating international trade and financial flows. Since the middle of the 20th century, the US dollar has played this role. A very large proportion of international exchange in currencies has the US dollar on one side of the transaction (Bank of International Settlements, 2010). In this sense the dollar acts as a '*vehicle currency*', i.e., a medium of exchange between currencies. Up until the second world war however, the British pound was the most accepted international currency, and before that, in the seventeenth and eighteenth centuries, it was the Dutch guilder.[1] In this paper, we develop a dynamic general equilibrium model of a vehicle currency to formally address the following questions concerning a vehicle currency: What are the gains to a vehicle currency? How are these gains distributed among the participating countries? How do changes in monetary policy or monetary arrangements induce countries to switch from one vehicle currency to another?

In frictionless models of international trade there is no reason for exchange between countries to take place in any particular currency. In practice, however, transactions costs of trading lead agents to make and receive payments in a currency which has a high trade volume and is widely acceptable to all countries. It is cheaper for payments between agents in small countries with thinly traded currencies to be made indirectly using US dollars than to use direct bilateral trade in their own currency markets. While there are clear welfare benefits to a vehicle currency in avoiding transactions costs of multiple currency trade, it introduces an asymmetry into the international monetary system by giving a central role to one currency. This may give the residents of the country issuing that currency an advantage, either in the ease with which payments may be made, or through the direct gains from issuing a currency which is in demand by residents of other countries.

By their nature, vehicle currencies are likely to become locked-in in a way which gives the issuer of the currency a natural monopoly. On the other hand, the historical record shows that the international system does switch from one international currency to another. In contemporary debate, one might ask whether the vehicle currency role of the dollar will be lost in favor of the euro. This is likely to depend on the configuration of US economic policies. The option of using alternative currencies as vehicles surely places some constraints on the actions of monetary and fiscal authorities of vehicle currency countries.

The economics literature has long recognized the benefits of a vehicle currency as a solution to a problem of transactions costs (e.g. Krugman, 1980, Black, 1991). But this literature has almost wholly been either simply descriptive, or based on partial equilibrium models in which

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[1]Eichengreen (2011) gives a detailed historical account of the rise of the dollar as an international currency.

relative prices or trades are exogenous. There are few general equilibrium models analyzing the way in which a vehicle currency facilitates international exchange (see below for references). In the absence of a general equilibrium framework, it is not possible to assess the efficiency gains to a vehicle currency, nor to address the nature of the asymmetry inherent in such a system, or the limits on economic policies that are necessary to maintain the role of a vehicle currency.

This paper develops a dynamic general equilibrium model of a vehicle currency. In our model, a vehicle currency arises as an equilibrium outcome in the manner described narratively above. That is, by eliminating costly bilateral exchange in small currency markets, a vehicle currency can reduce the transactions cost of exchange. But the advantage of a fully specified general equilibrium model is that we can be precise about the trading mechanism underlying a vehicle currency equilibrium, the effect of a vehicle currency on equilibrium exchange rates, and the nature and magnitude of gains to a vehicle currency. In addition, we can use the model to analyze the specific gains to the issuer of such a currency. Finally, we can explore how a vehicle currency arises, and the constraints on monetary policy necessary for a vehicle currency to survive.

We build a monetary exchange economy model with N countries, where $N \geq 3$. The money of a particular country is required to finance purchases in that country, as a result of a cash-in-advance constraint. But agents can choose the way in which they acquire foreign currencies. We model foreign exchange trade as a costly process that takes place through 'trading post' technologies, as in Shapley and Shubik (1977), Starr (2000) and Howitt (2005). They represent locations where agents can go in order to buy or sell one currency for another; that is, they facilitate bilateral trade in currencies. But trading posts are costly to set up. In a purely symmetric equilibrium, there would be one trading post for each possible bilateral pair of currencies. Trading possibilities would be the same for the holders of any currency, so that currencies and countries would be treated equally. But in a world with a large number of currencies, this environment would involve significant real resources being used up in setting up trading posts. An alternative equilibrium is where one country operates as the medium of exchange between currencies, i.e., as a vehicle currency. The vehicle currency offers significant efficiencies, since less resources are used up in trading. At the same time however, it confers significant benefits on the vehicle currency issuer. The main object of the paper is to explore these gains and their distribution.

More precisely, in a Symmetric Trading Equilibrium (STE), there are $N(N-1)/2$ bilateral foreign exchange trading posts, and agents from any country can use their currency directly to buy the currency of any other country. In a Vehicle Currency Equilibrium (VCE), country 1 acts as an intermediary. There are only $N-1$ trading posts, with currency 1 being on one side of all currency trades. Agents from any country $i > 1$ who wish to purchase currency $j \notin \{i, 1\}$ must

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first purchase currency 1 and then use currency 1 to purchase currency j.

The gains to a VCE come from being able to facilitate all possible trades while reducing the number of trading posts by $(N/2 - 1)(N - 1)$. For large N, these gains may be substantial. The gains are reflected in smaller bid-ask spreads in currency markets. But the gains are unevenly distributed. Residents of the issuing country have the same opportunity set as in an STE, since they can directly buy the currency of any other country. But residents of the peripheral countries (i.e. all countries $i > 1$) must visit two trading posts in order to complete an exchange with another peripheral country. This imposes additional costs of trade. We find that a VCE always benefits residents of country 1. But residents of peripheral countries may lose or gain.

The model points to three key features in the assessment of the gains to a vehicle currency. The first is the number of currencies. The more independent countries and currencies, the greater are the transactions gains to using a vehicle currency in exchange. With only a small number of currencies, a vehicle currency will not offer much welfare gain for peripheral countries, because the costs of indirect exchange will offset the gains to reduced transactions costs for peripheral countries. The second key feature is the size of countries. Larger countries have a natural advantage as providers of the vehicle currency because they engage in more international trade than smaller countries, leading to larger volume in foreign exchange markets involving their currency. Finally, the monetary policy followed by the authority of the vehicle currency is a crucial determinant of the size and distribution of the gains to a vehicle currency. A higher rate of inflation in the vehicle country shifts the transactions gains away from the rest of the world, and towards vehicle currency residents. But if the vehicle country is large, the use of a vehicle currency may still offer substantial benefits, even with quite a high rate of inflation. There is a natural trade-off between size and inflation.

We also provide a quantitative analysis of the size of the welfare gains to a vehicle currency. We do this by inferring the transactions costs involved in currency exchange from the observed spreads in foreign currency trade. If we use the spreads typically used in 'corporate customer' markets in foreign exchange, the gains can be very large - over one percent of GDP for the vehicle currency country, and about half of this for the other countries. Alternatively, a calibration based on spreads from the foreign exchange interbank market, where spreads are extremely narrow, suggests that there still remain gains to a vehicle currency, but they are much smaller for all countries.

We also use the model to explore the degree to which a vehicle currency is sustainable. Because the model combines fixed costs and 'network externalities', there are many Nash equilibria of the conventional type that are robust to deviation by individual agents. In order to explore the

robustness of a VCE we investigate the incentives for deviation by aggregate groups of agents. We show that the robustness of a vehicle currency depends in very intuitive way on the three features just described. There is a three-way trade-off between monetary policy, country size, and the number of currencies that are required to prevent peripheral countries from deviating from the VCE. We show that the introduction of a single currency area among peripheral countries (such as the euro) tends to significantly tighten the constraints imposed on a vehicle currency in order to maintain robustness of the VCE. This is because a single currency area simultaneously reduces the number of existing currencies, reducing the transactions costs gains to a vehicle currency, and increases the economic size of the area issuing a peripheral currency. Both of these effects tend to work together to discipline the money growth rate of the vehicle currency.

There is a relatively small literature on international currencies. Krugman (1980) defines a vehicle currency in the same way that is used here, within a partial equilibrium setting, and explores alternative trading patterns. An important paper by Rey (2001) examines how increasing returns to scale technologies in financial markets may give rise to an international currency. In Rey's model, there are three countries, each of which is fully specialized in producing a perishable commodity, issuing one currency and issuing one-period discount bonds denominated in the domestic currency. There is a cash-in-advance constraint on the goods market, with goods paid for in the currency of the seller. Consumers cannot engage in foreign exchange operations themselves but have to request the services of a financial intermediary. Intermediaries operate a currency exchange technology that has constant returns to scale at the level of individual intermediaries but increasing returns to scale at the aggregate level, so that the average cost of an intermediary falls as its labor input increases; in this sense there is a thick market externality.

Our analysis contrasts with Rey's in four main ways. First, the structure of the model, and the source of gains from a vehicle currency are entirely different. We model currency exchange as facilitated by trading posts rather than externality. With externalities, there is room for policy to internalize the externalities. With trading posts, the thinness or thickness of the market is reflected by prices and the bid-ask spread. Inefficiencies arise not from externalities but from a lack of coordination. Second, the three key elements in our model – country size, varying the number of countries, and the role of vehicle country monetary policy – are not analyzed by Rey. Third, we focus on welfare analysis, both qualitatively and quantitatively. We show how the gains and (potential) losses to a vehicle country are allocated, and we show the crucial role of the center country in the determination of these gains. Finally, we explore the robustness of the vehicle currency to deviations by individual countries, as well as to the inception of a 'rival' vehicle currency.

Another related paper by Hartmann (1998) looks at a model of a vehicle currency in financial markets and endogenizes a bid-ask spread. Despite the important role of a vehicle currency in financial markets, we choose to focus instead on the role of a vehicle currency in international trade, as Krugman (1980) and Rey (2001) do. This focus helps us to understand why pound sterling served as a vehicle currency when the British empire was the dominant player in international trade in the nineteenth century, and why the US dollar rose to a similar role after the US became the dominant player in international trade. Although it is theoretically possible that a currency like the Swiss Franc can serve as a vehicle currency, the historical evidence shows that being the dominant player in international trade has been a necessary condition for a country's currency to become a vehicle currency.

A different literature on search and money has explored the use of international currencies in an environment where agents can choose the currency they will hold to make purchases (e.g. Matsuyama et al., 1993, Shi, 1995, Zhou, 1997, and Wright and Trejos, 2001). Although it is desirable to use such a framework to provide a microfoundation for the coexistence between local/domestic and international currencies, we do not use the framework here. The main reason is that it is difficult to generalize the coexistence of local currencies with an international currency – Without the technical assumptions in these models, such as indivisible money and take-it-or-leave-it offers by buyers, there is a tendency for all currencies to end up circulating as international currencies. Since our focus is on a vehicle currency as a medium of exchange between currencies, rather than a medium of exchange between goods, we assume cash-in-advance constraints to give each country's currency a special role in the trade for the country's goods. One can view this assumption as a result of a legal restriction on settlement with domestic currency within a domestic market.[2]

The theoretical model of the paper is built around the assumption that one currency acts as a 'vehicle' in international trade. Empirical evidence of this role for the US dollar is widely available. Goldberg and Tille (2005) establish that the dollar is overwhelmingly used for invoicing both export and import prices for the US economy. Even for non-US related exports, they note that a substantial component is invoiced in US dollars. McKinnon and Schnabl (2004) note that much of intra-Asian trade is invoiced and settled in US dollars. More detailed evidence for US dollar invoicing and trade payments in Asia is provided in Cook and Devereux (2006). In addition, it is well known that exports of primary commodities are substantially invoiced in US dollars.

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[2]Head and Shi (2003) and Liu and Shi (2010) construct search-based models of two countries in which goods trade for money, and monies also trade for one another. Liu (2011) calibrates a stochastic version of a similar model to quantitatively evaluate the importance of productivity and monetary shocks in explaining the volatility in exchange rates and other macro variables.

Goldberg and Tille (2005) show dollar invoicing of commodities is a much more predominant phenomenon even than dollar invoicing of trade in non-primary commodities. Devereux, Shi and Xu (2010) find that of 81 raw material price series published by the UNCTAD, only 5 are not dollar denominated. Similarly, 30 out of 35 commodity contracts in the Rogers International Commodities Index are written in US dollars. Finally, Table A1 in the Appendix presents some stylized facts regarding US dollar invoicing in overall trade flows for selected countries.[3]

It is important to clarify that the paper does not mean to construct a model of the foreign exchange market. As is well recognized, currency trade in the foreign exchange market is many orders of magnitude greater than the value of world trade. In our model, currency trade takes place only in order to finance international trade. Eichengreen (2011) notes the importance of the US economy as a world trading nation in the adoption of the US dollar as an international currency during the post World War II period. From a modeling perspective, it is hard to measure the welfare benefits of the gross trade in foreign currencies that takes place in current foreign exchange markets. On the other hand, to the extent that an international currency facilitates trade in goods, the welfare benefits are more easily quantified. This is the emphasis of our paper.

The paper is organized as follows. Section 2 develops the basic model. Section 3 analyzes the STE, and Section 4 the VCE. Section 5 quantitatively assesses the comparison between the VCE and the STE. Section 6 explores the robustness of the VCE. Some conclusions then follow.

2. The Model

2.1. Technology and preferences

Time is discrete, indexed by $t = 0, 1, ...$ There are $N \geq 3$ countries, indexed by $i = 1, 2, ..., N$. The world population is normalized to unity. Country i has population n_i, so that $\Sigma_{i=1}^{N} n_i = 1$. We call n_i the size of country i. The world economy has a continuum of goods of measure one. Country i is endowed with measure n_i of these types of goods, with each resident being endowed with one unit of a particular type of good. Thus, the endowment per capita is the same across countries (i.e., 1).[4] All goods are perishable at the end of a period.

Within a country, all households are alike. Let $c_{ij}(k)$ denote a country i resident's consumption of good k produced by country j, and $u(c_{ij}(k))$ be the utility of such consumption. Because all

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[3]Although this empirical literature on invoicing currencies is indicative of the use of vehicle currencies in practice, we should caution that the two concepts are different theoretically. A vehicle currency in the sense defined in our paper, and in Krugman (1980), is a medium of exchange between other currencies. Although an invoice currency is very likely to act as such a medium of exchange in practice, it does not have to in theory.

[4]This modeling of country size and endowments allows us to vary the size of a country without affecting the endowment per capita (or "productivity") of that country. In the calibration of the model in section 5 below, we will allow for non-traded goods endowments, so as to more accurately quantify the gains to a vehicle currency. The qualitative results of section 3 and 4 are unaffected by this.

goods endowed to a country are symmetric, $c_{ij}(k) = c_{ij}$ for all k produced by country j. Thus, a country i household's total utility in a period from consuming country j goods is $\int u(c_{ij}(k))dk = n_j u(c_{ij})$. Such a household has the following intertemporal utility function:

$$U^i = \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{N} n_j u(c_{ijt}),$$

where $\beta \in (0, 1)$ is the discount factor. Throughout the analysis, we will assume that $u(c) = \ln(c)$.

Until the end of section 6, we assume that each country has its own currency, and its residents receive lump-sum transfers only from their own country's monetary authority. Let M_i be the total stock of currency i per capita in the world. Let m_{ji} be the stock of currency i held by a country j household, *normalized by* M_i. If country i residents hold all their own currency, then symmetry within a country implies that $m_{ii} = 1/n_i$. The gross rate of growth of currency i is defined as $\gamma_i = M_{i(+1)}/M_i$, where the subscript $+1$ indicates the next period. Proceeds from money growth are transferred to domestic households. Let $\tau_i M_{i(+1)}$ be the amount of transfers to each country i household. That is, $M_i + n_i \tau_i M_{i(+1)} = M_{i(+1)}$, which implies that

$$\tau_i = (\gamma_i - 1) / (\gamma_i n_i).$$

For convenience, we normalize all nominal quantities and prices measured in currency i by the total stock of currency i, i.e., by M_i.

2.2. Monetary exchange at trading posts

Purchases of country i's goods must use only currency i. Because of this cash-in-advance constraint, a household in country i must obtain currency j in order to consume country j's good. The purpose of imposing this constraint is to focus on the exchange between currencies, rather than between currencies and goods, as discussed in the Introduction.

Currency trade is organized in bilateral trading posts. At a trading post, one currency is exchanged for another. We order the two currencies at a post in ascending order and refer to a trading post with currencies k and j as post kj, where $k < j$. There cannot be instantaneous arbitrage between trading posts or shorting on a currency.[5]

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[5]The use of trading posts allows us a simplified way to handle the frictions inherent in currency trading. In reality of course, currency traders do not just trade one currency for another. But there are clear limits on the number of exchange possibilities that exist. Few commercial currency exchanges are willing to buy or sell much more than about a half dozen currencies. One reason is that it is costly to hold inventories of currencies that are not often in demand. Bid ask spreads are typically higher for more thinly traded currencies. Figure A1 in the Appendix shows a clear negative relationship between inter-bank spreads and size of bilateral foreign exchange trade for a number of the most widely traded currencies. It would be possible to extend the model to allow any one trading post to trade multiple currencies, so long as there were additional costs associated allowing for additional currencies to be handled at each post. The evidence from spreads suggests that this is approximately true.

Operating a trading post involves a fixed cost. In order to operate trading post kj, the manager of a trading post must incur a fixed cost ϕ in both goods k and j. There is also a cash-in-advance constraint on trading posts - the fixed cost in each country's good needs to be paid in that country's money. Examples of this fixed cost include the wage cost of workers who operate the post and the amortized amount of the initial cost of setting up the post. For simplicity, we abstract from the flexible cost that depends positively on the trading volume at the post.

Trading posts are a contestable market (see Tirole, 1988, p308). That is, anyone can set up a trading post and offer prices for the exchange between two currencies, but only one successful manager will run a trading post with zero net profit. The manager of each trading post announces two prices for a pairwise trade, one for the sale of a currency (ask) for another currency, and one for the purchase (bid) of a currency for another currency. Under the assumption of contestable markets, there is Bertrand competition among managers at the stage of entering the market (see Howitt, 2005, for a similar formulation). Thus, the manager of a trading post surviving the competition offers the bid and ask prices that are just sufficient to cover the fixed costs of setting up the trading post, given the buyers and sellers of the currency pair in which the trading post operates. These prices then represent the equilibrium nominal exchange rates for each currency pair. Thus, nominal exchange rates in this formulation will reflect not just standard 'macro fundamentals', but also the fixed costs of currency trade.

Let f_{ik}^{kj} be the amount of currency k (normalized by the total stock of currency k) brought to the post kj by the representative country i household. Because households cannot short on currencies at any post, $f_{ik}^{kj} \geq 0$ for all i, k, j. The post kj is said to be active if at least one side of the post has a positive amount of currency, i.e., if $\sum_{i=1}^{N} f_{ik}^{kj} + \sum_{i=1}^{N} f_{ij}^{kj} > 0$.

With N countries and trading posts for each pair of currencies, there are $N(N-1)/2$ possible trading posts. But with each trading post incurring fixed costs, in principle this can be improved upon by using one currency as an intermediate, and trading twice, buying the intermediate, or 'vehicle' currency, and then selling it to obtain the currency required for purchasing the desired goods. When one currency plays the role of a 'vehicle', then only $N-1$ trading posts need to exist in order to facilitate trade between all countries.

With fixed costs of setting up trading posts, there can be many Nash equilibria that differ from each other in the number of active posts. To see this, suppose that an agent believes that no (or only a few) other agents will go to a particular trading post. Then trading at that post will not be sufficient to cover the fixed cost, and so the agent will have no incentive to bring a currency to buy or sell at that trading post. In this case, the trading post will remain inactive.

2.3. Timing of events

The timing of events is as follows. At the beginning of a period, agents receive unspent cash balances in each currency. They receive their income from last period sales of their endowment, in their own currency, plus a currency transfer from their domestic monetary authority. At this point, money holdings are measured, and the balance of currency j held by a country i household is denoted m_{ij}. Agents then visit the trading posts of their choice in order to exchange currencies. After currency exchange at trading posts, the balance of currency j held by a country i household is denoted m'_{ij}. After the currency trading is over, they visit the goods market, with each household dividing into a shopper and a seller. At the end of the period, the households consume all the goods purchased. We will suppress the time subscript t whenever possible and use the subscript $\pm z$ to stand for $t \pm z$, where $z \geq 1$.

3. Symmetric Trading Equilibrium

Assume that there is a trading post open for every pair of currencies. In total, there are $N(N-1)/2$ posts open. Households of each country can then engage in direct currency trade in order to obtain the currency required to purchase any country's good. We describe an equilibrium of this setup as a Symmetric Trading Equilibrium (STE).

3.1. Household choices

Consider an arbitrary country i and let us examine the decision problem of a representative household in country i. For given money holdings, the household chooses a sequence $\{h_{it}\}_{t=0}^{\infty}$. For each period t, the vector h_{it} consists of the following choices: consumption of the goods produced by country j, c_{ij}, the portfolio of currencies traded at the currency posts, $(f_{ii}^{ij})_{j>i}$ and $(f_{ii}^{ji})_{j<i}$, the portfolio of currencies held immediately after trading in the currency market, $(m'_{ij})_{j=1}^{N}$, and the portfolio of currencies held after receiving monetary transfers in the next period, $(m_{ij(+1)})_{j=1}^{N}$. The household's choices maximize U^i subject to the following constraints:

$$m_{ii} = \frac{1}{\gamma_i}\left[m'_{ii(-1)} - n_i p_{i(-1)} c_{ii(-1)} + p_{i(-1)}\right] + \tau_i \tag{3.1}$$

$$m_{ij} = \frac{1}{\gamma_j}\left[m'_{ij(-1)} - n_j p_{j(-1)} c_{ij(-1)}\right], \quad j \neq i, \tag{3.2}$$

$$m'_{ii} = m_{ii} - \sum_{j>i} f_{ii}^{ij} - \sum_{j<i} f_{ii}^{ji}, \tag{3.3}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{ij}^a} f_{ii}^{ij}, \quad i < j, \tag{3.4}$$

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$$m'_{ij} = m_{ij} + s^b_{ji} f^{ji}_{ii}, \quad i > j, \tag{3.5}$$

$$m'_{ij} \geq n_j p_j c_{ij}, \text{all } j. \tag{3.6}$$

The nominal exchange rate s^a_{kj} is the currency k 'ask' price of currency j, i.e., the amount of currency k (normalized by the total stock of currency k) that a household must supply to the post kj in order to obtain one unit of currency j (normalized by the total stock of currency j).[6] Likewise, the nominal exchange rate s^b_{kj} is the currency k 'bid' price of currency j, i.e., the amount of currency k (normalized by the total stock of currency k) that a household can obtain at the post kj with one unit of currency j (normalized by total stock of currency j). Clearly, $s^a_{kj} \geq s^b_{kj}$ is required for trading post kj to be viable.

Equation (3.1) describes the dynamics of domestic cash balances and (3.2) the dynamics of the balances of foreign currencies. For the domestic currency, holdings at the beginning of the period consist of left-over currency in the last period, sales of goods in the last period, and monetary transfers. Note that the household spends $n_i p_i c_{ii}$ on all domestic goods (where p_i is the normalized price of good i), but receives income only from its own endowment p_i. Money growth γ_i is applied to the money carried over from the last period because $m'_{ii(-1)}$ and $p_{i(-1)}$ are normalized by last period's money stock. For a foreign currency $j \neq i$, holdings at the beginning of the period consist entirely of the left-over currency in the last period, as described in (3.2).

The household then visits the $N - 1$ currency trading posts, supplying the amount f^{ij}_{ii} of currency i at post ij for each $j > i$ and the amount f^{ji}_{ii} at post ji for each $j < i$. (Recall that we label a currency post kj with $k < j$.) After the currency exchange, the household's balance of currency i at the end of the period is given by (3.3). At the ij trading post $(j > i)$, the household pays the 'ask' price for currency j, and receives f^{ij}_{ii}/s^a_{ij} units of currency j in return. At the ji trading post $(j < i)$, the household receives the 'bid' price for its sale of currency i, and gets $s^b_{ji} f^{ji}_{ii}$ units of currency j. Hence, the household's holdings of currency j $(j \neq i)$ are described in (3.4) and (3.5). In addition, the cash-in-advance constraint (3.6) must be satisfied for all consumption of each country's goods.

We first examine the optimal choices of households, taking exchange rates as given, and then look at equilibrium exchange rates which ensure that trading posts are viable in an STE. To proceed, assume that all cash-in-advance constraints are binding.[7] This means that households have no foreign currency left over at the beginning of a period, and they hold the entire stock of

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[6]The normalization implies multiplying the typical nominal exchange rate by the currency j money stock, and dividing by the currency k money stock. This means that permanent differences in money growth across countries k and j do not affect s_{kj}.

[7]This outcome arises in the equilibrium provided that the gross rate of money growth of each currency is higher than the discount factor.

domestic currency. That is, $m_{ij} = 0$ for all $j \neq i$ and so $m_{ii} = 1/n_i$. The households must visit all trading posts in order to ensure that they can consume all goods.

The household's optimal choices can be described intuitively as follows. First, because the utility function is logarithmic and all goods appear symmetrically in a household's utility function, the optimal level of expenditure is the same on all goods. That is, the following equations hold (see Appendix A for a derivation):

$$\text{for } j > i: \quad s_{ij}^a p_j c_{ij} = p_i c_{ii}, \tag{3.7}$$

$$\text{for } j < i: \quad p_j c_{ij} = s_{ji}^b p_i c_{ii}. \tag{3.8}$$

Second, because the household holds no foreign currency across periods, consumption of a foreign good j must be financed entirely by the amount of currency j that the household purchases in the current period. That is, $f_{ii}^{ij}/s_{ij}^a = n_j p_j c_{ij}$ for $j > i$, and $f_{ii}^{ji} s_{ji}^b = n_j p_j c_{ij}$ for $j < i$. Third, all purchases of foreign currencies in the period must come from holdings of domestic currency at the beginning of the period. Therefore, using (3.3) together with these conditions, we get:

$$\frac{1}{n_i} = m_{ii} = n_i p_i c_{ii} + \sum_{j>i} s_{ij}^a n_j p_j c_{ij} + \sum_{j<i} \frac{1}{s_{ji}^b} n_j p_j c_{ij}. \tag{3.9}$$

Substituting the first-order conditions for consumption into (3.9), we have:

$$p_i c_{ii} = \frac{1}{n_i}, \tag{3.10}$$

$$f_{ii}^{ij} = \frac{n_j}{n_i}, j > i; \quad f_{ii}^{ji} = \frac{n_j}{n_i}, j < i. \tag{3.11}$$

A household brings more of its total cash balances to trading posts that offer the currency of larger countries, because a larger country has more types of goods that the household needs.

3.2. Trading posts and exchange rate determination

There is a firm at each trading post ij. The firm sets prices s_{ij}^a and s_{ij}^b so as to just break even, after it incurs the fixed cost ϕ in good i and ϕ in good j. The firm must pay these fixed costs with currency. Hence, the firm must hold currency i in the (normalized) amount $p_i \phi$ and currency j in the (normalized) amount $p_j \phi$. As a result, exchange rates in trading post ij must satisfy two conditions. The first condition, determining the ask price of currency j, is:

$$s_{ij}^a \left[n_j f_{jj}^{ij} - p_j \phi \right] = n_i f_{ii}^{ij}, \tag{3.12}$$

This is explained as follows. In an STE, trading post ij receives total currency j payments of $n_j f_{jj}^{ij}$ (since only country j agents hold currency j at the beginning of each period in this

equilibrium), and must hold currency $p_j\phi$ to pay the good j fixed costs of setting up the trading post. It receives $n_i f_{ii}^{ij}$ deliveries of currency i from country i residents. It must set the ask price of currency j that country i residents will pay so that its holdings of currency j, in excess of its fixed costs, are all paid out to country i households. This condition is given by (3.12).

In a similar manner, to determine the bid price, s_{ij}^b, the trading post must satisfy the condition that deliveries of currency i made by country i households, less required currency holdings of $p_i\phi_i$, must equal the deliveries of currency j by country j residents. This condition is:

$$s_{ij}^b n_j f_{jj}^{ij} = n_i f_{ii}^{ij} - p_i\phi. \tag{3.13}$$

From the fact that all cash-in-advance constraints bind, in conjunction with market clearing, we have that $m_i = 1 = n_i p_i$, so that $p_i = 1/n_i$, for all i. Using this in (3.12) and (3.13), and substituting the solutions for the currency trades f_{ii}^{ij}, we get:

$$s_{ij}^{aSTE} = \frac{n_j}{n_i - \phi/n_j}, \quad s_{ij}^{bSTE} = \frac{n_j - \phi/n_i}{n_i}, \text{ for } i < j. \tag{3.14}$$

Here, as in other places, we add STE to the subscripts of a variable to emphasize that the quantity of the variable is associated with a symmetric trading equilibrium. The above result shows that bilateral (normalized) nominal exchange rates are proportional to the relative size of the countries, adjusted for transactions costs. The bigger is country j relative to i, the greater is the total demand for currency j by country i residents, leading to a higher cost of j. We have to impose the restriction $\phi < n_i n_j$, for all i, j, so that these solutions are meaningful.

It is evident that the bid-ask spread at trading post ij under the STE is:

$$\left(s_{ij}^a/s_{ij}^b\right)^{STE} = \left(1 - \frac{\phi}{n_i n_j}\right)^{-2} > 1. \tag{3.15}$$

The equilibrium bid-ask spread reflects the presence of trading costs. The bid-ask spread will be higher, the smaller the countries i and j, since this implies that a smaller volume of total currency is brought by both buyers and sellers to the ij trading post.

From (3.14), (3.7) and (3.8), we find that consumption levels under the STE are:

$$c_{ii} = 1. \tag{3.16}$$

$$c_{ij} = 1 - \frac{\phi}{n_i n_j}, \text{ all } j \neq i. \tag{3.17}$$

Of each type of good endowed to a country i, a domestic resident of the country consumes one unit, and so total consumption of this good by domestic residents is n_i (< 1). In contrast, of each type of good endowed to a foreign country j ($\neq i$), a resident of country i consumes less than one unit and so total consumption of each foreign good by country i residents is less than

n_i. The presence of trading costs in the currency market introduces an endogenous home bias in consumption. Given the form of preferences and the trading cost technology, the STE has the property that the fixed costs of setting up the ij trading post are fully borne by households of country i and j. The fixed costs in terms of good j (i) are borne by country i (j).[8]

How does country size affect the outcome of the STE? From (3.17) above, we see that consumption is higher if the trade involves a larger country. Take the example where $n_1 = n$, and $n_i = n'$ for all $i > 1$. Since $\sum_i n_i = 1$ we must have $n' = (1 - n)/(N - 1)$. Consider the case where $n_1 > n'$. Hence, $c_{1i} = c_{i1} = 1 - \frac{\phi}{n_1 n'}$ for $i > 1$. In addition, $c_{1i} > c_{ij}$ for all $i, j > 1$. Consumption is higher if the trade involves a larger country. Intuitively, c_{1j} is higher than c_{ij}, because country 1 has more residents sharing the fixed good j cost of setting up trading post $1j$ than country i has to share the fixed good j cost of post ij. Likewise, c_{i1} is higher than c_{ij} because the good 1 fixed cost of setting up trading post $1i$ is spread among more goods than the good j cost of setting up trading post ij (or ji, if $i > j$). In this example, since $c_{1j} > c_{ij}$ for all $i, j > 1$, we may also conclude that country 1 residents have higher welfare than other countries. Because of its size, country 1 receives higher consumption of all other country's goods, whereas all other countries receive higher consumption of only country 1's good.

Note that consumption in the STE is independent of home or foreign country money growth. Money is neutral, and there are no international 'spillovers' of monetary policy.

Finally, we check that the cash-in-advance constraints indeed bind. Using the first order conditions above, it is easy to establish that cash-in-advance constraints for each currency i will bind in a steady state if $\gamma_i > \beta$.

4. Currency 1 as a Vehicle

Now assume that currency 1 serves as the vehicle currency. In a VCE (Vehicle Currency Equilibrium) currency 1 has active trading posts with all other currencies, but there are no bilateral posts except those with currency 1. This reduces the total number of trading posts from $N(N-1)/2$ to $N - 1$. We call country 1 the *VC country* or the center country and other countries the *peripheral countries*. Most derivations and proofs for this section are in Appendix A.

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[8]To see how this is consistent with market clearing, note that for each individual good in country j there is an amount $1 - (N - 1)\phi/n_j$ available for consumption, which is equal to the endowment less the cost of setting up $N-1$ trading posts, averaged over the number of goods in the country. Total consumption is $\sum_{j=1}^{N} n_i c_{ij}$. Substituting the solutions for consumption above, it can be established that this equals the available endowment.

4.1. Household decisions

In a VCE, residents of all other countries $i > 1$ must engage in two foreign exchange transactions in order to consume goods other than their own or country 1's good. This means that from the time of their decision to consume an additional unit of these goods, they must wait one period for consumption to take place. To obtain other peripheral country currencies $j \neq i, 1$, a household in a peripheral country i ($\neq 1$) must carry a positive amount of the vehicle currency between periods. That is, $m_{i1} > 0$ for all $i \neq 1$. As a result, the total holdings of currency 1 by country 1 residents must be lower than the entire stock of currency 1, i.e., $m_{11} < 1/n_1$. Because the peripheral countries hold currency 1 between two adjacent periods, the cash-in-advance constraint on currency 1 does not bind for these countries. In contrast, for the VC country, the cash-in-advance constraint on currency 1 binds under the same conditions as in the STE. Also, as before, the cash-in-advance constraints on all non-vehicle currencies bind for all countries. Thus, $m_{ij} = 0$ for all $i \neq j$ and $j \neq 1$, and $m_{ii} = 1/n_i$ for all $i \neq 1$[9] .

The decision problem facing country 1 is the same as before, because country 1 has active trading posts with all other countries. For country $i > 1$, the dynamics of money holdings are still given by (3.1) and (3.2), and the cash-in-advance constraints by (3.6). However, the other constraints are modified as follows:

$$m'_{ii} = m_{ii} - f^{1i}_{ii}, \tag{4.1}$$

$$m'_{i1} = m_{i1} - \sum_{j \notin \{i,1\}} f^{1j}_{i1} + s^b_{1i} f^{1i}_{ii}, \tag{4.2}$$

$$m'_{ij} = m_{ij} + \frac{1}{s^a_{1j}} f^{1j}_{i1}, \quad j \notin \{i,1\}, \tag{4.3}$$

$$m_{i1} \geq \sum_{j \neq i} f^{1j}_{i1}. \tag{4.4}$$

Constraint (4.1) says that the only domestic currency i that the household spends in the currency market is that brought to the $1i$ post. The household's holding of the vehicle currency coming out of the foreign exchange market is described by (4.2). This comprises its initial holding of vehicle currency m_{i1}, less its purchases of other peripheral currencies, made with vehicle currency, i.e. $\sum_{j \notin \{i,1\}} f^{1j}_{i1}$, plus new purchases of vehicle currency, $s^b_{1i} f^{1i}_{ii}$. The constraint

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[9]In a previous version of the paper, we also examined a variation of the model in which each household can go through two rounds of trading at the currency posts in a period. In the first round, they trade the domestic currency for the VC and, in the second round, they trade the VC for other peripheral currencies. Although there are some minor differences, most of the comparisons between a VCE and an STE in that variation of the model are similar to what we will report here.

(4.3) gives the household's holdings of other non-vehicle currency $j \notin \{1, i\}$ after the currency exchange. The household uses the vehicle currency to exchange for such a non-vehicle currency at the $1j$ post, and the amount of the vehicle currency that the household brings to the post is f_{i1}^{1j}. Finally, (4.4) requires that the total amount of the vehicle currency that the household brings into the ij posts should not exceed the amount that the household has when it enters the period. We may call this constraint the 'vehicle currency constraint'. It prevents the household from short sales in vehicle currency, since $m_{i1} \geq 0$ must always hold. The vehicle currency constraint binds, provided $\gamma_1 > \beta$.

To see how the use of a vehicle currency affects a peripheral country's consumption choices, we derive the following conditions for a peripheral country i (> 1):

$$p_i c_{ii} = \frac{1}{s_{1i}^b} p_1 c_{i1} \tag{4.5}$$

$$s_{1i}^b p_i c_{ii} = \frac{\gamma_{1(+1)} s_{1j(+1)}^a}{\beta} p_{j(+1)} c_{ij(+1)}, \ j \notin \{i, 1\}. \tag{4.6}$$

The condition (4.5) characterizes the trade-off between consuming good 1 and the domestic good, which is the same as before. But the trade-off involved between consumption of the domestic good and another peripheral country good is quite different, as shown by (4.6). Sacrificing one unit of the domestic good gives p_i in domestic currency, and hence $s_{1i}^b p_i$ in currency 1 when converted at the $1i$ trading post. This can only be converted into a country j's ($j \notin \{i, 1\}$) currency in next period's foreign exchange trading session. In the next period, each dollar of currency 1 can obtain $1/[\gamma_{1(+1)} s_{1j(+1)}^a p_{j(+1)}]$ units of good j. Equating the costs and benefits in utility terms, and discounting, gives condition (4.6).

Thus, the use of a vehicle currency affects a peripheral country's optimal consumption of other peripheral countries' goods in three aspects, relative to the STE. First, to consume other peripheral goods, they must undertake two foreign exchange transactions, accepting the bid price of their own currency i in terms of currency 1, and paying the ask price of currency $j \notin \{1, i\}$ in terms of currency 1. Second, the transaction involves a delay, which is costly because agents discount future utility. Finally, it also involves a cost due to country 1 money growth, as country 1 inflation will reduce the real value of their currency 1 money holdings over time.

As in the previous section, only residents of country $i \neq 1$ hold currency i between periods. Thus, $m_{ii} = 1/n_i$ and $p_i = 1/n_i$ for all $i \neq 1$, as before. Also, a country i's holdings of currency i are equal to the sum of expenditures on goods. However, the condition (3.9) needs to be modified to take into account that the expenditures on other peripheral countries' goods occur with a one period delay, as explained above. Accordingly, for a peripheral country i, the use of a vehicle

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currency changes the country's consumption of its domestic good, its supply of and demand for currencies, and its holdings of the vehicle currency as follows:

$$c_{ii} = \frac{1}{\delta_i}, \tag{4.7}$$

$$f_{ii}^{1i} = \frac{1}{n_i} - n_i p_i c_{ii} = \frac{1}{n_i} - \frac{1}{\delta_i}, \tag{4.8}$$

$$f_{i1}^{1j} = \frac{\beta n_j}{n_i \delta_i} \left(\frac{s_{1i(-1)}^b}{\gamma_1} \right), \quad j \notin \{i, 1\}, \tag{4.9}$$

$$m_{i1} = \sum_{j \notin \{i,1\}} f_{i1}^{1j} = \frac{\beta(1 - n_i - n_1)}{n_i \delta_i} \frac{s_{1i(-1)}^b}{\gamma_1}. \tag{4.10}$$

where $\delta_i \in (\beta, 1)$ is defined as

$$\delta_i \equiv n_i + n_1 + \beta(1 - n_i - n_1).$$

Let us summarize the differences between (4.7) – (4.10) and the counterparts in the STE. First, (4.7) shows that for $\beta < 1$, a peripheral country consumes a higher share of its own good than under STE, since trading off consumption of the domestic good for another peripheral country's good involves waiting one period, and future consumption is discounted. Second, the household in a peripheral country i brings a larger volume of the domestic currency to the $1i$ trading post under VCE than under STE (see (4.8)), because the household brings to the $1i$ post all of its currency that remains after spending on the domestic good. Third, in the VCE, a peripheral country's household brings currency 1 to exchange for another peripheral country's currency, rather than using the domestic currency to exchange for another peripheral country's currency. The amount of currency 1 used in such exchange by a country i, given by (4.9), depends on discounting, country 1 money growth, and the previous period's bid rate at which currency i was sold. For all $\beta \leq 1$ and $\gamma_1 \geq 1$, we can establish that (4.9) is smaller than the quantity of currency i that the household brings to the ij post in the STE, which is n_j/n_i. Finally, (4.10) shows that the total amount of currency 1 that country i holds at the beginning of the period is positive (rather than zero as in the STE), and that it is affected by discounting, country 1 money growth and the previous period's bid price of currency i.

For country 1, optimal consumption is chosen in the same manner as under the STE and, so, the consumption levels are:

$$c_{11} = \frac{m_{11}}{p_1}, \quad c_{1i} = \frac{m_{11}}{s_{1i}^a p_i}, \text{ for } i \neq 1. \tag{4.11}$$

The amount of currency 1 brought to the $1i$ post by a country 1 household is:

$$f_{11}^{1i} = n_i m_{11}. \tag{4.12}$$

The household's balance of currency 1 at the beginning of a period is:

$$m_{11} = \frac{1}{n_1} \left(1 - \frac{\beta}{\gamma_1} \sum_{i \neq 1} \frac{(1 - n_i - n_1) s_{1i(-1)}^b}{\delta_i} \right). \tag{4.13}$$

This is less than the amount in the STE (i.e., $m_{11} < 1/n_1$) because, in the VCE, the peripheral countries also hold currency 1 in order to finance their consumption of other countries' goods. Note that this result implies that $f_{11}^{1i} < n_i/n_1$.

Finally, the price of country 1's good is different from that in the STE because currency 1 is used as a VC. Using the fact that the cash-in-advance constraint on currency 1 binds for country 1 and substituting $\tau_1 = (\gamma_1 - 1)/\gamma_1 n_1$, we rewrite the constraint (3.1) for $i = 1$ as follows:

$$n_1 p_1 = 1 - \gamma_{1(+1)} \left[1 - n_1 m_{11(+1)} \right]. \tag{4.14}$$

Thus, country 1's price level is influenced by the holdings of currency 1 by all other countries.

4.2. Trading posts with a vehicle currency

We now determine exchange rates under the VCE. In each period, country i residents in total bring $n_i f_{ii}^{1i}$ to the $1i$ post. At the $1i$ post, currency 1 is supplied by country 1, in the amount $n_1 f_{11}^{1i}$, and by each of the other peripheral countries $j \notin \{i, 1\}$, in the amount $n_j f_{j1}^{1i}$. Then, the ask and bid prices of currency i are determined by:

$$s_{1i}^a \left[n_i f_{ii}^{1i} - \phi p_i \right] = n_1 f_{11}^{1i} + \sum_{j \notin \{i,1\}} n_j f_{j1}^{1i}, \tag{4.15}$$

$$s_{1i}^b n_i f_{ii}^{1i} = n_1 f_{11}^{1i} + \sum_{j \notin \{i,1\}} n_j f_{j1}^{1i} - p_1 \phi. \tag{4.16}$$

We focus on a steady state where γ_1 is constant over time. Then, all real variables and all normalized nominal variables are constant over time. In the steady state, the above conditions in the currency market and the condition (4.13) yield the following proposition:

Proposition 4.1. *Under the VCE, ask and bid exchange rates for trading posts $1i$, $i > 1$, may be written as:*

$$s_{1i}^{bVCE} = \delta_i \left[D_i - (1 - n_1 m_{11}) E_i \right], \tag{4.17}$$

$$s_{1i}^{aVCE} = \delta_i \frac{(\delta_i - n_i) \left[D_i - (1 - n_1 m_{11}) E_i \right] + p_1 \phi}{\delta_i (1 - \phi/n_i) - n_i}, \tag{4.18}$$

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where D_i and E_i are functions of (n_i, n_1, γ_1) and given in Appendix A. The price level of the VC country's good is given by (4.14) and the money balance m_{11} obeys:

$$1 - n_1 m_{11} = \frac{\sum_{i \neq 1}(1 - n_i - n_1)D_i}{\gamma_1/\beta + \sum_{i \neq 1}(1 - n_i - n_1)E_i}.$$
(4.19)

The full expressions for s_{1i}^{bVCE} and s_{1i}^{aVCE} are quite complicated. In order to develop the intuition behind the solutions, we begin by focusing on some special cases.

4.3. Some special cases

Case A: $\{n = 1/N, \gamma_1 = 1, \beta \to 1\}$. In this case, all countries are of equal size, country 1 money growth is zero, and the discount factor tends to unity. For this case, the only difference in the opportunity set of a peripheral country's agents and the VC country's residents is that the former must engage in indirect trading. This special case enables us to focus on the difference between a VCE and an STE in the trade volume at the currency posts.

In this case, the solution for s^{bVCE} is:

$$s^{bVCE} = \frac{(1 - \phi N^2)}{(N - 2)(N - 1)(1 - \phi N^2)/N + 1}.$$
(4.20)

This exchange rate is lower than (3.14). Thus the VCE pushes down exchange rates for the peripheral countries. Both the demand for and supply of currency i at the trading post $1i$ rise in the VCE, relative to the STE. But demand rises by less than supply, since the increase in the demand for i by peripheral countries (bringing currency 1 from last period) is partly offset by a lower demand for i from the residents of country 1, the vehicle currency country, given that their money holdings are lower.

The value of s^{aVCE} in case A is:

$$s^{aVCE} = \frac{s^{bVCE}}{(1 - N^2\phi)\,\Omega_A\,(N)},$$
(4.21)

where

$$\Omega_A\,(N) = \frac{N - 1 - N^2\phi}{N - 1 - (N - 2)\,N^2\phi} > 1 - N^2\phi.$$

Comparing (4.21) with (3.15), we see that the bid-ask spread is lower in a VCE under the case A than in the STE, for all feasible values of ϕ. Intuitively, greater trade volume on both sides of the foreign exchange market pushes down spreads.

Case B: $\{n = 1/N\}$. This case is more general than Case A, and we use it to illustrate the effects of the VC country's money growth. While the case restricts all countries to be of equal

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size, it leaves the discount factor and the rate of country 1 money growth to be arbitrary. In this case, we can write the bid-ask spread as:

$$\left(\frac{s^a}{s^b}\right)^{VCE} = \frac{\Omega_B(\gamma_1)}{(1 - \phi_1 N^2)^2},$$

(4.22)

where

$$\Omega_B(\gamma_1) = \left(1 - \phi_1 N^2\right) \frac{\beta(N-2) + 1 - \beta(N-2)N\phi_1\left(N - 1 + \frac{1}{\gamma_1}\right)}{[\beta(N-2) + 2 - (\beta(N-2) + 2)N^2\phi]} < 1.$$

Again, the bid-ask spread is smaller than under STE.

However, the spread is increasing in the VC money growth. Higher country 1 money growth reduces a peripheral country's currency deliveries to each trading post in a VCE, thus reducing trading volume and bidding up spreads. But it is still the case that $\lim_{\gamma_1 \to \infty} \Omega_B(\gamma_1) < 1$. Money growth can not generate a spread higher than that in the STE.

4.4. Efficiency gains and resource allocation with a vehicle currency

The VCE reduces the resources needed to operate the exchange, relative to the STE, and hence raises available world resources for consumption. In the VCE, each peripheral country sets up just one trading post. With less resources used up in trading posts, there are more of all goods $i > 1$ available for consumption, and the same amount of good 1. For large N, this efficiency gain can be substantial. But at the same time, the vehicle currency introduces an asymmetry into the allocation of world resources. In this section, we analyze the nature of the global gains from a vehicle currency, as well as the asymmetric gains achieved by the vehicle currency country. Again, we begin with some special cases.

Case A: $\{n = 1/N, \gamma_1 = 1, \beta \to 1\}$

In this case, the efficiency gains from the VCE are easy to illustrate. In the STE, each country's net output of each of its goods is $1 - \phi N(N-1)$, which is the endowment less the cost of setting up $N - 1$ trading posts, divided by the number of goods in the country, $1/N$. In a VCE, net output of each center country good is unchanged, since it must set up $N - 1$ trading posts still. But net output of each good of each peripheral country is now $1 - \phi N$, since only one trading post is set up for each country.

The benefits of the VCE go disproportionately to the VC country, although net output of each peripheral country good is higher in a VCE than in an STE. For Case A, we may show that:

$$c_{11}^{VCE} = 1,$$

(4.23)

$$c_{1i}^{VCE} = \Omega_A(N) \geq 1, \; i > 1,$$

(4.24)

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where Ω_A is defined following (4.21). Country 1's consumption of the home good is the same as in STE. Consumption of all other country's goods differs from (3.17), however. It is easy to see that $c_{1i}^{VCE} > c_{1i}^{STE}$. Moreover, from (4.24), $\Omega_A(3) = 1$, and $\Omega'_A(N) > 0$, so that $c_{1i}^{VCE} \geq 1$. Since c_{11} is unchanged, and c_{1i} is higher, the VC country is unambiguously better off than in the STE.

For a peripheral country i, we can establish that:

$$c_{ii}^{VCE} = 1 \tag{4.25}$$

$$c_{i1}^{VCE} = (1 - \phi N^2) \tag{4.26}$$

$$c_{ij}^{VCE} = (1 - \phi N^2)\Omega_A(N). \tag{4.27}$$

These results reveal a few interesting contrasts of a peripheral country's consumption to the level in the STE and to the VC country's consumption in the VCE. First, a peripheral country's consumption levels of the domestic good and country 1 good are the same as in the STE, but consumption of *other* peripheral countries differs. Second, the gain from VCE for peripheral countries is lower than that of the VC country, because $c_{ij}^{VCE} < c_{1j}^{VCE}$ for all $i > 1$ and $i \neq j$ (see (4.27)). Third, in equilibrium, all the transactions costs of setting up trading posts are borne by the peripheral countries. The comparison between (4.26) and (4.23) reveals that the good 1 cost of setting up the $1i$ trading post is borne by country i. In addition, the comparison between (4.27) and (4.24) reveals that the good j cost of the $1j$ trading post is also borne by country i. In fact, since $\Omega_A(N) > 1$, for $N > 3$, the VC country consumes more than the average endowment of peripheral goods, so that in a VCE, the peripheral countries incur *more* than the full amount of the transactions costs.

Does this mean that peripheral countries are worse off? The answer is no, because, while they bear all the transactions costs, the overall transactions costs are far lower in VCE than in STE, and the transactions cost saving is increasing in the number of countries, N. From (4.27), we know that $c_{ij}^{VCE} \geq c_{ij}^{STE}$, with strict inequality for $N > 3$. Because $c_{ii}^{VCE} = c_{ii}^{STE}$ and $c_{i1}^{VCE} = c_{i1}^{STE}$, and for $N = 3$, $c_{ij}^{VCE} = c_{ij}^{STE}$, then for the case of three countries, peripheral countries are exactly as well off in VCE as in STE. But for $N > 3$, $c_{ij}^{VCE} > c_{ij}^{STE}$, and welfare is higher under VCE. The higher is N, the greater is the transaction cost saving due to the vehicle currency.

We can link the efficiency gains to changes in the terms of trade. For the VC country, the consumption level of a peripheral good is $c_{1i}^{VCE} = c_{11}p_1/(s_{1i}^a p_i)$. Since c_{11} is constant in the current special case, a rise in the consumption level c_{1i}^{VCE} is equivalent to country 1 receiving a higher terms of trade or a lower relative price of the peripheral good. Indeed, we can write $p_1/(s_{1i}^a p_i) = \Omega_A(N)$, which is greater than the analogous price under STE (i.e., $1 - \phi N^2$). For

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the peripheral countries, consumption of other peripheral country goods is written as $c_{ij}^{VCE} = c_{ii}s_{1i}^b p_i/(s_{1j}^a p_j)$. Since c_{ii} is constant, the increase in consumption of other peripheral country goods, relative to the STE, comes about only if there is a fall in their relative price, $(s_{1j}^a p_j)/s_{1i}^b p_i$. In case A, $s_{1i}^b p_i/(s_{1j}^a p_j) = (1 - \phi N^2)\Omega_A(N)$, which is also higher than the counterpart in the STE. Thus, the existence of a vehicle currency effectively improves the terms of trade for *all countries*. Nevertheless, the gains for country 1 exceed those for peripheral countries. Country 1 has to trade only once in order to consume any good, while peripheral countries must trade twice. Even without time discounting or money growth, this leads the terms of trade gains to be lower for the peripheral country, relative to the VC country. In addition, as we have noted, for $N = 3$, *all the gains* go to the VC country.

Case C: $\{\gamma_1 = 1, \beta \to 1, n_1 = n, n_i = (1 - n)/(N - 1), i > 1\}$.

We use this case to illustrate how the level and distribution of welfare gains from a VCE change with the VC country's size. In this case, country 1 can have a different size from peripheral countries. For instance, if $n > 1/N$, then $n_i < 1/N$ for all $i > 1$, which implies that the VC country is larger than all peripheral countries.

Two features in Case C are the same as in Case A. First, $c_{11}^{VCE} = c_{11}^{STE}$, $c_{ii}^{VCE} = c_{ii}^{STE}$, and $c_{i1}^{VCE} = c_{i1}^{STE}$ for all $i > 1$, and so the VCE does not change a country's consumption of its domestic good or a peripheral country's consumption of the VC country's good. Put differently, the vehicle currency only makes a difference for consumption of peripheral country goods for country 1, and consumption of non-domestic peripheral goods for the countries $i > 1$. Second, the use of currency 1 as the vehicle currency enables country 1's households to obtain higher consumption of peripheral countries' goods than under the STE. That is, $c_{1j}^{VCE} > c_{1j}^{STE}$ for all $j > 1$, whatever the VC country's size.

In contrast to Case A, the peripheral countries do not always gain. To see this, we derive:

$$c_{ij}^{VCE} - c_{ij}^{STE} = \phi \rho_1 \left[n(N - 4 + 3n) - \phi(N - 1)(N - 3) \right], \tag{4.28}$$

where

$$\rho_1 \equiv \frac{[(N - 1)/(1 - n)]^2}{n(n + N - 2)(1 - \phi\rho)} > 0 \quad \text{and} \quad \rho \equiv \frac{(N - 2)(N - 1)}{n(n + N - 2)} < 1.$$

It is possible to have $c_{ij}^{VCE} < c_{ij}^{STE}$, in which case peripheral countries must lose as a result of the VCE. Take the case $N = 3$ as an example. In this example, the expression inside the square parentheses in (4.28) is $n(3n - 1)$ and, hence, $c_{ij}^{VCE} < c_{ij}^{STE}$ if $n < 1/3$. This result contrasts with Case A, where $c_{ij}^{VCE} = c_{ij}^{STE}$ when $N = 3$. The contrast is intuitive. In Case A with $N = 3$, the peripheral countries were indifferent between the VC and STE. The costs of indirect trade

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were just offset by the gains from shutting down trading posts. But in Case C with $N = 3$ and $n < 1/3$, the costs of indirect trade exceed the gains from fewer trading posts, since using the vehicle currency involves trading through a smaller market with higher transactions costs. Thus, a VCE where the vehicle currency country is smaller than the average sized country may reduce welfare for peripheral countries.

We may also explore the way in which the gains from the VCE change in response to changes in country size. These responses are given as:

$$\left.\frac{d(c_{1j}^{VCE} - c_{1j}^{STE})}{dn}\right|_{n=1/N} < 0, \quad \left.\frac{d(c_{ij}^{VCE} - c_{ij}^{STE})}{dn}\right|_{n=1/N} > 0. \tag{4.29}$$

Thus, the consumption gains for the VC country are negatively related to its size. In the STE, a rise in country 1's size has a large effect on country 1's consumption of all goods $j > 1$, as it reduces the per capita cost of a trading post, reducing the exchange rate country 1 residents must pay. But in the VCE, the increase in country 1's size has a smaller impact, because each trading post has more currency j on the other side. A marginal increase in the size of the vehicle currency economy has a diluted impact on its exchange rate.

In contrast, for peripheral countries, the gain goes in the opposite direction. A rise in the relative size of country 1 will reduce c_{ij}^{STE}, since each peripheral country becomes relatively smaller. But in the VCE, the negative impact of a rise in n is diminished, because country i is purchasing country j's good via the $1i$ and $1j$ currency markets. Hence, while the VC country size tends to lower gains for the VC country itself, it will raise gains for peripheral countries.

Case D: $\{\beta \to 1, n = n_i = 1/N\}$.

We use this case to examine the impact of the VC country's money growth, γ_1, again assuming very low time discounting, and all countries being of equal size. We can verify the following effects. First, country 1's consumption of every good increases in γ_1. That is, c_{11}^{VCE} and c_{1j}^{VCE} increase in γ_1 for all j. Since, under STE, allocations are independent of monetary policy, clearly the gains to VCE for country 1 are increasing in γ_1. Second, country 1 money growth reduces peripheral country consumption of both good 1 and all other peripheral country goods. In the extreme case where $\gamma_1 \to \infty$, a peripheral country's consumption of another peripheral country's good goes to zero and, hence, consumption of a peripheral country's good goes only to residents of that country and country 1.

Money growth in the VC affects allocations as above because it represents a tax on peripheral country holders of the VC. Inflation in the VC progressively erodes the usefulness of the VC in exchange. Although this effect is easy to understand, we should note that a peripheral country's

consumption of the VC country's good also decreases in the VC money growth rate, despite the fact that the financing for consumption of good 1 does not require peripheral country residents to hold currency 1 over time. This happens because higher money growth reduces the demand for currency $i > 1$ coming from residents of all other peripheral countries, since it reduces the value of these agents currency 1 holdings. This pushes down the exchange rate that country i residents receive in the $1i$ trading post, reducing their terms of trade. In this way, money growth has both a direct and an indirect effect on peripheral country welfare.

Case D assumes $\beta \to 1$. However, the results just illustrated hold for general $\beta \leq 1$, as we state in the following proposition (see Appendix A for a proof).

Proposition 4.2. *Under the assumption that $n_i = n = 1/N$, $i > 1$, the VCE satisfies the following features: (i) s_{1i}^a/s_{1i}^b is increasing in γ_1; (ii) c_{i1} ($i \neq 1$) is decreasing in γ_1, but c_{ii} is independent of γ_1; (iii) c_{ij} ($j \neq i, 1$) is decreasing in γ_1; (iv) c_{11} is increasing in γ_1; (v) c_{1i} is increasing in γ_1.*

5. A Quantitative Welfare Comparison

Now move on to the general model, taking into account money growth, country size, time discounting, and variation in the number of countries. We wish to examine the welfare gains from a vehicle currency, relative to the STE. For this section, we again assume that all peripheral countries are of equal size, so that $n_i = n/(N-1)$, for all $i = 2, ..., N$. We make one additional change to the model specification. In the model above, we assumed that all goods are internationally traded. Quantitatively, this implies that countries are much more open to trade than observed in international trade data. We handle this by amending the preference and technology to allow for a non-traded goods sector, assuming that, as for traded goods, non-traded goods are exogenously endowed to residents of each country. By varying the size of the non-traded goods sector we can vary the trade to GDP ratio in each country. This extension makes a difference only for the welfare estimates of the gains from a vehicle currency, as described more fully below.[10]

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[10]The amended preference specification is now

$$U^i = \sum_{t=0}^{\infty} \beta^t \left[\alpha_i u_n(c_{int}) + (1 - \alpha_i) \sum_{j=1}^{N} n_j u(c_{ijt}) \right],$$

where c_{int} represents consumption of non-traded goods. The higher is α_i, the share of non-traded goods in utility, ceteris paribus, the smaller will be the gains from a vehicle currency, since these gains apply to consumption of a narrower range of goods. Note that the presence of non-traded goods have no effect on the *qualitative* results of previous sections, because the extensions generates only a transformation of utility and there is no substitutability in production between sectors. But α_i is clearly important for an estimate of the gains from a VC.

Note that the model is then quite parsimonious - there are only six parameters to choose; β, γ, n, N, α (the share of non-traded goods in preferences) and ϕ.

We calibrate the model as follows. The first parameter to choose is β, the discount factor. In the special cases above we let $\beta \to 1$ essentially assuming no discount factor, so that the period is arbitrarily short. Although it is reasonable to assume that the carrying time period of a vehicle currency is very short, the function of a vehicle currency extends across a number of different frequencies. So the choice of β should represent a compromise between different perspectives on the use of a vehicle currency. For some financial traders, the holding period of currency might be hours or days, while for other exporters or importers using vehicle currency to facilitate ongoing transactions, the time period may be significantly longer. More generally, the need to hold either vehicle currency cash or liquid assets in order to facilitate trade might impose a cost over a much longer horizon. Since our analysis is focused on currency use for commodity trade, we use a quarterly frequency setting $\beta = 0.99$. At first glance, it might seem that the welfare results would be substantially affected by the choice of β, with the benefits to the VC country substantially magnified by the need for peripheral country residents to hold the vehicle currency for a longer duration. This is not the case, because the fixed cost of the trading posts has to be incurred in each period no matter how short a period is. In fact, the choice over β is critical only insofar as it influences the way in which VC country inflation rates affect peripheral countries. The higher is β, the less the impact of VC inflation on peripheral countries, since with very short period money holdings, the inflation tax imposed by VC money growth has little impact on peripheral country income. Intuitively, if β is calibrated to a very high frequency (e.g. weekly or daily), even extremely high annual inflation rates of the VC country would have little impact on the use of the vehicle currency. This may be somewhat unrealistic. We expect that the use of an international currency is clearly going to be eroded by looser monetary policies of the VC country. Hence, our choice of β is made in an attempt to generate a more realistic link between VC country inflation rates and the acceptability of the vehicle currency. It is important to emphasize however, (as is clear also from the previous section) that the asymmetric welfare impacts of the vehicle currency on the VC relative to the peripheral countries is *not* solely tied to the inflation tax, or to the calibration of β. Even with $\beta = 1$ and $\gamma = 1$, the VC country generally gains disproportionately from the VC equilibrium due to its role as the mediator of currency transactions. This message comes through clearly in the numerical results below.

With the quarterly frequency for β, the value of the gross money growth rate γ_1 is taken from the US CPI growth rate over 1980-2011, which was 0.5 percent at a quarterly frequency. Thus we set $\gamma_1 = 1.005$.

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The overwhelming majority of currency trade in world financial markets is accounted for the big 'G10' currencies, given by the US dollar, the euro, the yen, the UK pound, the Swiss Franc, the Canadian dollar, the Australian dollar, the Swedish krona, the Singapore dollar, and the Norwegian krone.[11] We thus set $N = 10$ in the baseline model. But we also allow for variations from the baseline in the numerical results below. We allow for variations in n, the share of VC country GDP, although a baseline case for this would be $n = 0.2$, approximately the share of the US in world GDP.

The key parameter for the welfare estimates of a VC is ϕ, the transaction cost of setting up a currency trading post. Indirect estimates of transactions costs can be obtained from observed bid-ask spreads (e.g. Black, 1990, Boothe, 1988, Glassman, 1987) in foreign exchange markets. Bid-ask spreads in foreign exchange markets depend critically on the market being studied (see Beckaert and Hodrick, 2011, Lyons, 2001). In the foreign exchange interbank market, where large financial trades in currencies take place (and which accounts for the bulk of total foreign exchange trading), spreads are extremely low - in the order of 2-5 'pips', meaning digits in the fourth decimal place of the currency quotes (Lyons, 2001). Using published spreads in the interbank market for the G10 currencies (see http://www.canadianforex.ca/cgi-bin/interbank-spot-rates.asp), this translates into percentage spreads of 0.02 to 0.05 percent. However, the interbank foreign exchange market is arguably not the relevant real world equivalent to our model. As reported by the Bank for International Settlements (2010), daily trade in the interbank foreign exchange market is close to 4 trillion US dollars, many orders of magnitude greater than world daily trade flows. In our model, currency exchange takes place only to finance goods trade. Hence, the total volume of foreign exchange transactions is much lower than observed in the data. Moreover, in the interbank foreign exchange markets, most trading does not involve delivery of currency, but is netted out by banks and financial institutions. Since no inventory of currency needs to be held, transactions costs are extremely low. Moreover, trading positions are taken on an extremely short term horizon - most for less than a day (Lyons, 2001).

The most relevant foreign exchange market appropriate for our model is the 'customer market'. This is the market in which banks and foreign exchange brokers carry out foreign exchange trading with individuals and corporations, and requires actual delivery of foreign currency. The need to hold inventory, among other costs, leads to spreads in the customer market substantially higher than the interbank market, in the order of 1-1.5 percent for the major currencies and substantially larger for thinly traded currencies. Within the customer market however, there are different submarkets for individuals and corporations, with large corporations receiving more favorable

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[11]See http://marquezcomelab.blogspot.com/2007/10/list-of-g10-currencies.html.

spreads than individuals. Since most international trade is carried out by corporations, we take an estimate of the average quoted spread in the customer market for corporate clients.[12] This gives an average spread of 0.5.[13] Given the other parameters, this requires a value of $\phi = 0.0002$. We also allow for variations in ϕ below however.[14]

Finally, we follow convention in assuming that the non-traded goods share of GDP is 50 percent in all countries. This is an approximate average for OECD countries.[15]

We compare the allocations received under the VCE with those of the STE. As a welfare measure we compute the uniform percentage increase in the consumption of all goods that an agent would require, in the STE, to make her indifferent between the STE and the VCE. We denote this percentage change as dc_i, and compute this separately for agents of country 1 and country $j > 1$.

The vertical axis in Figure 1 represents the consumption benefit of the VCE, dc_i, for country 1, and for the representative peripheral country, for a range of values for the size of the VC country (the horizontal axis). For the baseline calibration with $N = 10$ and $n = 0.2$, the welfare gains to a vehicle currency are heavily weighted towards the center country. It gains the equivalent of 1.8 percent of consumption, while the peripheral country gains represent slightly less than 0.5 percent of consumption. But the gains are very sensitive to country size. If the center country is larger then the welfare gains are much closer for the centre and peripheral countries. At $n = .37$ the gains are equalized at 0.7 percent of consumption for both center and peripheral countries. As n rises above this, the peripheral countries gains more than the VC countries.

To account for the welfare benefits, note that in the STE, output of each good in country i is $(1 - \phi(N-1)N)$. For the calibration used in Figures 1 and 2, this implies that trading costs reduce output by 1.8 percent. By contrast, in the VCE, for a peripheral country, only one trading post must be formed. Output per good then is $(1 - \phi N)$, and transactions costs reduce output by only 0.2 percent. Even though individual transactions costs are very small, the overall cost can be very large when summed across a large number of bilateral trading posts. The aggregate welfare benefits are then obviously tied directly to the size of ϕ and the number of countries.

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[12]In the US, half of international trade is intra-firm. This may reduce the need for currency exchange. But even with intra-firm trade, costs must be paid in local currency, so that at some point, export earnings in one currency must go to recoup costs in other currencies.

[13]See http://www.whichwaytopay.com/compare-foreign-exchange-corporate-summary.asp. This website gives a range of quoted spreads for corporate clients from 0.1% to 1.0%. The average spread is 0.5%.

[14]It is also important to note that spreads in foreign currency markets have been substantially falling over time, with the growth of internet brokerages, e.g. Beckaert and Hodrick (2011). Thus, the estimates of the welfare impact of a vehicle currency based on contemporary spreads will necessarily be substantially smaller than estimates based on historical spreads.

[15]See Stockman and Tesar (1995). This implies that the trade to GDP ratio is at most 50 percent for any country in our model. For the larger VC country, the calibration implies that trade to GDP is at most 30 percent.

The gains of the VC for country 1 are tied to the fact that it trades at a lower exchange rate in a VC than in the STE. But the higher is n, the smaller is the gap between the two exchange rates. For the peripheral countries, it is the opposite, since the larger is n, the greater the exchange rate gain from trading indirectly through the VC relative to bilateral trade with another peripheral country.

Figure 2 illustrates the welfare gains as a function of the number of countries, N, at the baseline estimates for all parameters, and fixing $n = 0.2$. From the analytical results above, we know that peripheral countries do not benefit at all if there is zero discounting, zero money growth, $n = 1/N$, and $N = 3$. Here we see that with positive VC country money growth and discounting, peripheral countries are *worse* off in a VCE for low values of N. Thus, in the baseline calibration, peripheral countries only gain from a VCE if N is above a critical level. For Figure 2, a vehicle country is beneficial to the peripheral countries only for $N \geq 7$. But then as N rises above this, the welfare gains rise substantially. We see also that for the peripheral countries, the gains may not be monotonic in N. For small N, an increase in the number of countries makes each country more open. This means that in the VCE, a peripheral country is more exposed to the inflation tax of country 1, while in the STE such inflation has no effect. Hence, beginning at $N = 3$, an increase in N may reduce welfare for a peripheral country initially, relative to STE. But as N rises further, the benefits of reduced transactions costs take over, and the gains are increasing in N.

How high can γ_1 increase before it eliminates the gains for the peripheral countries? This will depend upon both N and n. For a large number of countries, and a VC country which is large relative to others, there are still gains to a vehicle currency even for high rates of VC money growth. Figure 3 shows the relationship between γ_1 and n such that the peripheral countries are just indifferent between the VCE and STE. Peripheral countries are strictly better off in the VCE than in the STE if γ_1 and n take values in the area below the curve that represents this relationship. When $n = 0.2$, peripheral gains from the VC are eliminated at $\gamma_1 = 1.018$. But if n is very high, so the VC country has a large share of world GDP, then VC inflation rates can be much higher before eliminating the welfare gains to a vehicle currency.

5.1. An alternative calibration

The calibration above is based on implied transactions cost inferred from spreads in the customer market for foreign exchange. In addition, we took a broad view of the type of frictions involved in the use and exchange of a vehicle currency, assuming that the average period over which actions take place was a quarter. What happens when we take an alternative calibration, based more

specifically on more high frequency foreign exchange trades? As we noted, the interbank market for foreign exchange has much narrower spreads than does the customer market. Here we explore the gains to a vehicle currency when costs are inferred from these lower spreads. In Figure 4 we present an alternative calibration of the model, assuming now that the transactions costs are inferred from inter-bank spreads. We assume a spread of 5 basis points, or 0.05 percent. Since this case obviously pertains to very high frequency trading, we set $\gamma = 1$, and focus on the case where $\beta \to 1$, so periods are arbitrarily short, and there is no inflation tax penalty at all in the use of the vehicle currency. Matching a 0.05 percent ask-bid spread at the value of $n = 0.2$ yields $\phi = 0.00002$, one tenth of the value in the baseline calibration. In this case, the gains to a vehicle currency are reduced greatly, although there are still positive gains for both VC and peripheral countries. At a value of $n = 0.2$, both gain roughly 0.07 of a percent of permanent consumption.

6. Robustness of the Vehicle Currency Equilibrium

We have shown that there may be large welfare gains to a VCE. However, there can be many equilibria which differ in the currency that serves as the VC. This multiplicity is inevitable when there are fixed costs of organizing currency exchange. If some bilateral markets are not open, then no individual trading firm has an incentive to incur a fixed cost in order to trade in that market, since, with no customers, it will perceive that there are no profits to be gained. In the presence of multiple equilibria, how can a particular currency arise to play the role of a vehicle currency? In this section, we try to shed light on this issue by addressing a closely related question: Once a currency has been serving as the VC, how can its VC role be robust to potential competition? We examine the following competitive forces on the VC: a deviation by two countries to trading their currencies directly, the formation of a currency block such as the euro, and an alternative vehicle currency.

Note also that any equilibrium with a particular VC is robust to the refinement of trembling hands by a small measure of agents or of evolutionary stability.[16] In order for a deviation from any equilibrium to have aggregate consequences, it must be undertaken by a large number of agents. In this section, such "large deviations" are undertaken by all agents within one country or a group of countries. One way to think of this national deviation is as an implicit policy choice by national governments.

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[16]For example, if a small measure of agents from any two countries exchange their domestic currencies directly in the VCE constructed above, they will make a loss as the amount of currencies brought into that post will not be sufficient to cover the fixed trading cost. Similarly, if a small measure of agents deviate to using a different currency as the vehicle currency, they will make a loss also.

6.1. Bilateral deviations

Consider a bilateral deviation by two countries, say, countries 2 and 3. Suppose that all households in the two countries trade their own two currencies directly. Other countries do not participate in the 23 post. Moreover, countries 2 and 3 still supply their domestic currencies to trade for currency 1 and use currency 1 to get other peripheral currencies. However, country 2 does not use currency 1 to buy currency 3, and country 3 does not use currency 1 to buy currency 2.

Denote $I = \{1, 2, 3\}$. For a country $i \notin I$, the decision problem is the same as in the VCE before, because all currency posts which the country participated before are still active after the above deviation. Since the decision problems of a household in country 2 and of a household in country 3 are images of one another, we only formulate the problem for country 2.

With the deviation, a household in country 2 faces the following constraints involving currencies 1, 2 and 3:

$$m'_{22} = m_{22} - f_{22}^{12} - f_{22}^{23}, \quad m'_{23} = m_{23} + \frac{1}{s_{23}^a} f_{22}^{23},$$

$$m'_{2j} = m_{2j} + \frac{1}{s_{1j}^a} f_{21}^{1j}, \ j \notin I, \quad m'_{21} = m_{21} - \sum_{j \notin I} f_{21}^{1j} + s_{12}^b f_{22}^{12},$$

$$\sum_{j \notin I} f_{21}^{1j} \leq m_{21}.$$

Other constraints that the household faces, such as the cash-in-advance constraints in the goods markets, are the same as those in the previous section. Because country 2 still needs currency 1 to exchange for other non-I currencies, the cash-in-advance constraint on currency 1 in the goods market does not bind for country 2, as in the previous section. All other cash-in-advance constraints bind. One can characterize the household's optimal decisions (see Appendix B).

At the 23 post, bid/ask prices satisfy $f_{22}^{23}/s_{23}^a = f_{33}^{23} - \frac{\phi}{n_3}$ and $s_{23}^b f_{33}^{23} = f_{22}^{23} - \frac{\phi}{n_2}$. Substituting the quantities of currencies traded (see Appendix B), we obtain the following bid and ask prices at the 23 post:

$$s_{23}^b = \left(\frac{n_3}{\delta_2^d} - \frac{\phi}{n_2} \right) \left(\frac{n_2}{\delta_2^d} \right)^{-1}, \quad s_{23}^a = \left(\frac{n_2}{\delta_2^d} - \frac{\phi}{n_3} \right)^{-1} \frac{n_3}{\delta_2^d}, \tag{6.1}$$

where $\delta_2^d = n_2 + n_1 + n_3 + \beta(1 - n_1 - n_2 - n_3)$. The bid-ask spread at the 23 post is smaller than that in the STE, provided $N > 3$. This is because, when $\beta < 1$, countries 2 and 3 will assign a higher fraction of their budget to each other's good than they will to other peripheral country goods, given that the consumption of those other goods requires a delay in consumption.

The deviation changes the equilibrium conditions for the 12 and 13 posts, because country 2 and country 3 do not supply all their residual balance of the domestic currency after purchasing

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the domestic good to the posts to exchange for currency 1. Similarly, the equilibrium condition for the $1j$ post, where $j \notin I$, also needs to be modified. We specify these conditions in Appendix B. In the analysis below, $j \notin I$ unless it is specified otherwise.

Is the deviation profitable for countries 2 and 3? In general, in order to assess this question we need to compare utility levels in a deviating equilibrium, relative to the VCE. But in the special case where $\beta \to 1$ and $\gamma_1 = 1$, a bilateral deviation by countries 2 and 3 leaves *unchanged* both the relative prices and consumption of all goods $i \notin \{2, 3\}$ by all countries $i = 1, ..., N$. This means that in assessing the benefits from a deviation to a bilateral trade for countries 2 and 3, we can simply look at the change in consumption of goods 2 and 3. Moreover, from (6.1), note that evaluated at $\beta = 1$, the bilateral exchange rates between currencies 2 and 3 are identical to those in the STE. This means that in the case $\beta \to 1$, and $\gamma_1 = 1$, $c_{23}^{DEV} = c_{23}^{STE}$. This implies that the conditions under which a bilateral deviation by countries 2 and 3 is beneficial to these countries are *equivalent* to the conditions that welfare of the peripheral countries under VCE is lower than that under STE (again in case $\beta \to 1$, and $\gamma_1 = 1$).

We may summarize the above results in the following proposition:

Proposition 6.1. *In the case $\beta \to 1$ and $\gamma_1 = 1$, there are no gains to deviating to a bilateral trading arrangement when $n \geq 1/N$. When in addition to $n \geq 1/N$, $N > 3$, the deviating countries are strictly worse off.*

Again, we note that the condition $n \geq 1/N$ may fail when n is too small, for the same reason that the VCE may lead to lower welfare than under STE. In addition, the result implies that, under this case, when considering a bilateral deviation, each country's welfare calculation is exactly aligned with average welfare for all peripheral countries. A bilateral deviation is only desirable individually when it is desirable in the aggregate.[17]

In the more general case where $\beta < 1$ and $\gamma_1 \geq 1$, a bilateral deviation has implications for consumption of all goods. Moreover, individual incentives are no longer aligned with aggregate welfare. But even then, the main impact of a bilateral deviation is on the consumption of the goods of the deviating countries, by the deviating countries themselves, and if γ_1 is large, by the

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[17]To gain another perspective on the effect of a bilateral deviation, we can compare the direct exchange of currency 2 for currency 3 and the indirect exchange through the vehicle currency. With the direct exchange, a household in country 2 gets $1/s_{23}^a$ units of currency 3 for each unit of currency 2. With the indirect exchange, one unit of currency 2 returns s_{12}^b units of currency 1 in the current period, which the household can use to exchange for s_{12}^b/s_{13}^a next period. In the absence of discounting and money growth, the indirect exchange through the vehicle currency gives a higher payoff to a household in country 2 than the direct exchange if and only if $s_{12}^b/s_{13}^a > 1/s_{23}^a$, or $s_{23}^a s_{12}^b/s_{13}^a > 1$. It turns out that this condition holds if and only if the gain from VC is negative. This partial equilibrium assessment is appropriate only in the case where $\beta \to 1$ and $\gamma_1 = 1$. When these conditions do not apply, then the bilateral deviation will change bid-ask spreads on bilateral trades other than the 23 trade.

VC country, since in the latter case, a deviation implies that it loses some inflation tax revenue. For the deviating countries, the switch to bilateral trade reduces the inflation tax embodied in trade using the vehicle currency, and as a result, consumption of the deviating partners good may rise, so long as n is relatively small. But if country 1 is large enough, the benefit from avoiding the inflation tax is offset by the higher transactions costs of trading bilaterally, relative to going through the cheaper vehicle currency.

Figure 5 illustrates the welfare gains from remaining in VCE, relative to a bilateral deviation, for the deviating countries. This is compared to the general welfare gain from the VCE, relative to STE, as calculated in the previous section. Under the calibration behind Figure 5, we see that at the baseline case of $n = 0.2$, there is no gain to a deviation. But for n below 0.1, the two deviating countries will gain, even though there are at this value for n, there positive benefits to the VCE, relative to the STE. In this sense, the VCE is *not* robust to large deviations whenever there is an implicit inflation tax imposed on the holding of vehicle currency. Moreover, even if n is large, the threshold inflation rate to prevent a bilateral deviation is substantially lower that which ensures that peripheral countries are indifferent in utility between VCE and STE. The reason is that two countries individually can pursue a bilateral deviation and avoid the inflation tax in their mutual trade, without giving up the benefits of a vehicle currency in trading with all other countries. Thus, at any values of n and N, the maximum value of γ_1 that eliminates a bilateral deviation is smaller than the value that eliminates gains from a vehicle currency for all the peripheral countries together. Figure 3 contrasts the trade-off between γ_1 and n that just eliminates the incentive to undertake a bilateral deviation from VCE, and the analogous trade-off for values that just eliminate gains from a VCE to the peripheral countries (as discussed above). Take again the case where the VC country is 20 percent of world GDP. Then the peripheral countries still gain from the VC even for (quarterly) inflation rates of 1.7 percent, as noted above. But in order to avoid a bilateral deviation, inflation rates must be no higher than 0.7 percent.

6.2. The introduction of the Euro

So far, we assumed that each country had its own currency. But this is not true of the Eurozone. In our model, if some countries join a single currency area, then the number of currencies as measured by N will fall, and the economic size of the currency area will equal the sum of the measure of goods produced in the member countries. This will reduce the incentive for countries outside the Eurozone to use the dollar as a vehicle currency. In terms of the last subsection, it will increase the gains from a bilateral deviation by two such outside members.

To address this question, we take the same example as before, but assume that $K \geq 2$

countries, from $j = (N + 1 - K), ..., N$, join a single currency area, eliminating the transactions cost of monetary trade within the area. This reduces the number of separate currencies in the world from N to $N + 1 - K$, and increases the economic size of the $N + 1 - K$ currency area.[18]

We calibrate as before so that initially $N = 10$, and assume that all peripheral countries are of equal size $n_i = (1 - n)/(N - 1)$. Let $K = 3$, so that the number of currencies falls from 10 to 8, and the size of the 8th 'country' is now $3(1 - n)/(N - 1)$. Now we ask, what is the incentive for a peripheral country $i \notin \{1, N + 1 - K\}$ and for the $N + 1 - K$ currency area to undertake a bilateral deviation in order to trade currency directly with each other rather than indirectly, using currency 1 as the vehicle currency. This trade-off now differs for country i and country $N + 1 - K$, since they do not have equivalent incentives for a bilateral deviation. In both cases, the trade-off shifts dramatically downwards as shown in Figure 6, indicating that the creation of a single currency area substantially increases the incentive to engage in a bilateral deviation, both for remaining peripheral currencies, outside the new single currency area, and for the new single currency area itself. As in Figure 5, Figure 6 shows the gain from remaining in the VCE, relative to deviating to bilateral currency trade, for a peripheral country and for a member of the single currency area, as a function of the size of the VCE country. For n less than 0.4, this gain is negative. In other words, there is a gain from deviating from the VCE both for a member of the single currency area, and for a peripheral country, unless the vehicle currency country is 40 percent of world GDP. An equivalent interpretation is that the maximum rate of inflation that the VCE country can sustain without triggering a deviation, shifts sharply downwards. In comparison with Figure 3 where all peripheral countries are of equal size, we find that for $n = 0.2$, even a zero rate of inflation would not be enough to eliminate a deviation towards bilateral monetary trade with the currency union.

This example suggests that the dominance of a vehicle currency becomes severely threatened by the set up of a large single currency area among peripheral countries, both because it increases the economic size of non-VC currency economies, and because it reduces the total number of currencies in existence. In separate ways, both effects increase the incentive to abandon a vehicle currency.

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[18]In this analysis, we take the currency area as a given institution. The determination of the number and size of currency areas is a separate question that cannot be addressed without enhancing the model. This is because according to the assumptions made here, a currency area removes the transactions cost of monetary exchange, and for a peripheral country, there is no gain to having a separate currency. Thus, all countries $j = 2, ..., N$ would wish to join the currency area. One interpretation of the experiment here is that there are costs to set up a currency area that are not modeled explicitly, and that the only efficient currency area is that among the K countries. For instance, if governments had unpredictable spending demands which required seigniorage revenue, then they would have to balance the needs for funds against the reduced transactions costs from joining a currency area.

6.3. Choosing a vehicle currency

The calculations in previous two subsections can be interpreted as measures of the restrictions imposed on the monetary policy of the vehicle currency in order to avoid deviations among peripheral countries. If these conditions fail, then of course all countries would have an incentive for a bilateral deviation. In this subsection, we perform another robustness check on the VCE. We explore the consequences of a switch from one vehicle currency to a second vehicle currency, where the switch is undertaken jointly by all peripheral countries in unison. Robustness of a VC with respect to such a deviation puts a tighter upper bound on the VC money growth than does a bilateral deviation.

First ignore country size differences, and assume that $n_i = 1/N$ for all $i = 1, ..., N$. We wish to compare the welfare from one VCE with an alternative vehicle currency. Assume initially that currency N is the vehicle currency, and denote this equilibrium VCN. Now compare this with another equilibrium where another currency is chosen as the vehicle currency. Without loss of generality, assume this is currency 1, and denote this equilibrium as $VC1$. Since countries are identical in all respects except money growth rates, the only source of welfare difference between $VC1$ and VCN arises from differences in γ_1 and γ_N.

To compare the two equilibria, we recall the following properties of the VCE from the previous sections: when currency N is the vehicle currency i, c_{iN} $(i \neq N)$ is decreasing in γ_N, (ii) c_{ii} is independent of γ_N; and (iii) c_{ij} $(j \neq i, N)$ decreases in γ_N. Together these properties imply that all countries $j = 2, ..., N$ will gain from the switch to $VC1$ if and only if $\gamma_1 < \gamma_{N.}$. This follows because by property (i), $c_{i1}^{VC1} > c_{iN}^{VCN}$, and by property (iii) $c_{iN}^{VC1} > c_{i1}^{VCN}$ and $c_{ij}^{VC1} > c_{ij}^{VCN}$.

Now consider country 1. Fix a good produced in country $i \neq N, 1$. We have:

$$\frac{c_{1i}^{VC1}}{c_{1i}^{VCN}} > \frac{c_{1i}^{VC1}}{\left[c_{1i}^{VCN} \right]_{\gamma_N = \gamma_1}} = \frac{\gamma_1/\beta + (\gamma_1 - 1)(N - 2)\left[1 - (N - 1)N\phi\right]}{1 - N^2\phi}.$$

The inequality follows from property (iii) above, and the assumption $\gamma_N > \gamma_1$, while the equality follows from re-arranging terms. The last expression is increasing in γ_1 and it is greater than one when $\gamma_1 = 1$. Thus, $c_{1i}^{VC1}/c_{1i}^{VCN} > 1$ for all $\gamma_1 \geq 1$.

We may also verify that $c_{1N}^{VC1} > c_{1N}^{VCN}$ for all $\gamma_1 \geq 1$. However, it is not necessarily true that $c_{11}^{VC1} > c_{11}^{VCN}$, even when $\gamma_1 \geq 1$, since more of good 1 is used up in transactions costs in $VC1$ than in VCN. However, given the logarithmic utility function, country 1's utility gain from consumption of good N will offset any losses from consumption of good 1, when comparing $VC1$

with VCN. That is, $(c_{1N}c_{11})^{VC1} > (c_{1N}c_{11})^{VCN}$.[19] Given this discussion, we may conclude that when all countries have the same size, a vehicle currency is robust with respect to all peripheral countries' joint deviation only if it has the lowest money growth rate among all the currencies. We state this result in the following proposition:

Proposition 6.2. *Assume $n = 1/N$, and $\gamma_1 < \gamma_N$. Every peripheral country i ($\neq N, 1$) is strictly better off in $VC1$ than in VCN. If $\gamma_1 \geq 1$, then country 1 is strictly better off in $VC1$ than in VCN. Therefore, VCN is not robust to a joint deviation to $VC1$ by the peripheral countries together. On the other hand, if $\gamma_1 > \gamma_N$, then VCN is robust to the joint deviation.*

Thus, we again find that the option of deviating, where here it is a joint deviation to a new vehicle currency, may place tight restrictions on the monetary policy of the VC country required to ensure sustainability of the VCE.

6.4. Coexistence of vehicle currencies

So far we have discussed only cases where one vehicle currency was used by all peripheral countries. But there is some evidence of 'currency blocs', in which certain geographic regions adopt regionally dominant currencies for intra-regional trade, but use alternative currencies for inter-regional trade. For instance, EU countries not in the euro area have begun to trade with one another in euro (Papademos, 2006), while it is well known that in Asia, the US dollar is the widely accepted trade currency (McKinnon and Schnabl, 2003). Is it possible to have multiple vehicle currencies exist within the modeling structure here? We briefly discuss this by way of a simple example in which there are hypothetically two vehicle currencies, and define the sense in which both vehicle currencies can coexist.

Say that currencies 1 and 2 are both vehicle currencies. Assume that all trading posts $1i$ and $2i$, for $i = 3, ..., N$, are open. In addition, just to make the example easier, assume that all peripheral countries $i = 3, ..., N$ are of equal size. Since all trading posts between 1, 2, and all peripheral countries are open, a peripheral country i may obtain currency $j \neq i$ through the $1i$ and $1j$ posts, or the $2i$ and $2j$ posts. Then it is easy to see that generically, only one vehicle

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[19]To see this, note that

$$\frac{(c_{1N}c_{11})^{VC1}}{(c_{1N}c_{11})^{VCN}} > \frac{(c_{1N}c_{11})^{VC1}}{\left[(c_{1N}c_{11})^{VCN}\right]_{\gamma_N=\gamma_1}}$$

$$= \frac{\left[1+\beta(1-\frac{1}{\gamma_1})(N-2)(1-(N-1)N\phi)\right]^2}{\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi\right]} \frac{[\beta(N-2)+1-(\beta(N-2)+2)N\phi]}{\left[1-N^2\phi\right]}$$

The last expression is an increasing function of γ_1 and its value at $\gamma_1 = 1$ is greater than one. Thus, $(c_{1N}c_{11})^{VC1} > (c_{1N}c_{11})^{VCN}$ for all $\gamma_1 \geq 1$.

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currency will be used. This is because the choice of whether to use vehicle currency 1 or vehicle currency 2 depends on a comparison of the cost of obtaining currency j through currency 1, which is $\gamma_1 s_{1j}^a / s_{1i}^b$, relative to obtaining currency j through currency 2, which is $\gamma_2 s_{2j}^a / s_{2i}^b$. If $\gamma_1 s_{1j}^a / s_{1i}^b < \gamma_2 s_{2j}^a / s_{2i}^b$, then no peripheral countries will use currency 2 as a vehicle currency. If $\gamma_1 s_{1j}^a / s_{1i}^b > \gamma_2 s_{2j}^a / s_{2i}^b$, then the opposite applies. Hence, the coexistence of two vehicle countries can only be supported in the knife-edge case where $\gamma_1 s_{1j}^a / s_{1i}^b = \gamma_2 s_{2j}^a / s_{2i}^b$.

In this example, therefore, there can be only one vehicle currency, if we define a vehicle currency as one which has open trading posts with all other currencies. But it is possible to have 'local' vehicle currencies in the following sense. Take the example again where currencies 1 and 2 are vehicle currencies. Instead of all posts $1i$ and $2i$ being open, however, assume that currency 1 has active trading posts only with currencies $I_1 = \{3, ..., N/2\}$ in addition to the 12 post, (assuming N is even), while currency 2 has active posts only with currencies $I_2 = \{N/2+1, ..., N\}$ in addition to the 12 post. In this case agents in peripheral countries $i \in I_1$ use currency 1 to purchase currency $j \in I_1$, $j \neq 1, 2, i$, and similarly agents in I_2 will use currency 2 to obtain other peripheral currencies in I_2. But, since there are no trading posts $1i$, $i \in I_2$ or $2i$, $i \in I_1$, agents in peripheral countries must trade in *both* vehicle currencies in order to trade currencies between I_1 and I_2. For instance, in order for agent $i \in I_1$ to purchase goods of country $j \in I_2$, she must first purchase currency 1. Then, in the next period, she will trade currency 1 for currency 2 at the 12 trading post. Finally, in the period *after* that, she obtains currency j at the $2j$ trading post, and consumes good j.

Clearly this equilibrium with local vehicle currencies is robust to individual deviations, since there is only a single channel within which to affect all money trades. Hence, the two vehicle currencies can coexist so long as they do not overlap within regional sub-groupings. For brevity however, we defer a full analysis of this case to a future paper.

7. Conclusions

This paper has developed a model in which a globally acceptable currency can function as a medium of exchange among countries, facilitating international trade, and economizing on resources when trading currencies requires costly transactions technologies. By eliminating the need to set up bilateral currency trading posts among all possible countries, a vehicle country reduces the average cost of currency trade. But the cost savings are distributed unevenly, with the center country gaining disproportionately. With a small number of countries, peripheral countries will be worse off with a vehicle currency relative to a symmetric trading equilibrium. But the gains from a vehicle currency may be substantial when there are a large number of countries

and currencies, and when the center country is large relative to peripheral countries. Even with many countries, however, these gains are eroded by higher rates of inflation in the VC country. If inflation in the center country goes too high, then our robustness analysis suggests that the use of the vehicle currency will collapse and countries may adopt another currency with lower inflation as the vehicle currency.

The model could be extended in a number of ways. We could allow for uncertainty in money growth and output levels. In this case, the risk-hedging properties of a vehicle currency would be important, in addition to its exchange use. We could also do a more explicit welfare analysis of monetary policy, assuming a social planner that weights each countries utility and can make monetary transfers across countries. Finally, we could allow for government issue of debt and explore the interaction between government budget deficits and monetary policy in influencing the sustainability of a vehicle currency. We leave these issues for future research.

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Appendix

A. Derivations and Proofs for Sections 3 and 4

(1) The derivation for (3.7) and (3.8). Let the current-value Lagrangian multiplier be λ_{ii} for (3.1), λ_{ij} for (3.2), η_{ii} for (3.3), η_{ij} for (3.4) and (3.5), and ψ_{ij} for (3.6). With the logarithmic utility function, the first-order conditions for c_{ij} and m'_{ij} yield the following result for all i and j:

$$\frac{1}{p_j c_{ij}} = \frac{\beta}{\gamma_{j(+1)}} \lambda_{ij(+1)} + \psi_{ij} = \eta_{ij}, \tag{A.1}$$

where the subscript $+1$ indicates the next period. The first-order conditions for f_{ii}^{ij} ($i < j$) and f_{ii}^{ji} ($i > j$) yield:

$$\eta_{ii} = \frac{1}{s_{ij}^a} \eta_{ij} \ (i < j); \quad \eta_{ii} = s_{ji}^b \eta_{ij} \ (i > j). \tag{A.2}$$

Dividing (A.1) for $j \neq i$ by the condition for $j = i$, and using (A.2), we obtain (3.7) and (3.8).

    (2) The derivation for (4.5) and (4.6). Let the current-value Lagrangian multiplier be η_{ii} for (4.1), η_{ij} for (4.3), η_{i1} for (4.2), and μ_{i1} for (4.4). As in the STE, the multiplier is λ_{ii} for (3.1), λ_{ij} for (3.2), and ψ_{ij} for (3.6). It is easy to verify that the first-order conditions for c_{ij} and m'_{ij} yield the same equation, (A.1), as in the STE. The first-order conditions for f_{ii}^{1i} and f_{i1}^{1j} are:

$$\eta_{ii} = s_{1i}^b \eta_{i1}, \quad i \neq 1, \tag{A.3}$$

$$\eta_{i1} + \mu_{i1} = \eta_{ij}/s_{1j}^a, \quad j \neq i, 1. \tag{A.4}$$

The envelope conditions for m_{ij} are:

$$\lambda_{ij} = \eta_{ij} \ (j \neq 1); \qquad \lambda_{i1} = \eta_{i1} + \mu_{i1}. \tag{A.5}$$

Substituting the last condition into (A.4) yields $\eta_{ij} = s_{1j}^a \lambda_{i1}$ for all $j \neq i, 1$. Dividing (A.1) for $j = i$ by (A.1) for $j = 1$, and using (A.3), we obtain (4.5).

    To establish (4.6), we show that $\psi_{i1} = 0$ for all $i \neq 1$. Suppose, to the contrary, that $\psi_{i1} > 0$. Then, $m'_{i1} = n_1 p_1 c_{i1}$, and so $m_{i1(+1)} = 0$ by (3.2). With (4.4), this further implies $f_{i1(+1)}^{1j} = 0$ for all $j \neq i$. That is, the household will have no foreign currency in the next period. As a result, consumption of foreign goods will be zero. This is not optimal since the marginal utility of such consumption is infinite when consumption is zero.

    Since $\psi_{i1} = 0$, (A.1) implies $\lambda_{i1(+1)} = \eta_{i1} \gamma_{1(+1)}/\beta$. Then, for all $j \neq i, 1$, we have:

$$\eta_{ij(+1)} = s_{1j(+1)}^a \lambda_{i1(+1)} = \frac{\gamma_{1(+1)}}{\beta} s_{1j(+1)}^a \eta_{i1} = \frac{\gamma_{1(+1)}}{\beta} \left(\frac{s_{1j(+1)}^a}{s_{1i}^b} \right) \eta_{ii}.$$

The first equality comes from a result derived above, the second equality is obvious, and the last equality comes from (A.3). Now, dividing (A.1) for $j \neq i, 1$ in the next period by (A.1) for $j = i$ in the current period, and using the above condition, we get (4.6).

(3) The derivation for (4.7) − (4.10). Consider a household in a country $i \neq 1$. Notice that the household spends the domestic currency in the current period to acquire currency 1 and to purchase domestic goods. Part of currency 1 that the household acquires today is spent on country 1 goods. The rest will be spent in *the next period* to purchase other peripheral currencies which, in turn, will be spent on goods of these peripheral countries. Thus, the household's holdings of domestic currency at the beginning of the period, $m_{ii} = 1/n_i$, are equal to the sum of three types of expenditures of the household: the current expenditure on domestic goods, the current expenditure on country 1 goods, and the expenditure in the next period on goods of other peripheral countries. That is, the following equation holds:

$$m_{ii} = 1/n_i = n_i p_i c_{ii} + \frac{1}{s_{1i}^b} \left(n_1 p_1 c_{i1} + \sum_{j \neq 1, i} \gamma_{1(+1)} s_{1j(+1)} n_j p_{j(+1)} c_{ij(+1)} \right). \tag{A.6}$$

Substituting (4.5) and (4.6) into (A.6), we can solve c_{ii} as in (4.7) and then recover c_{ij} for all $j \neq i$. The result (4.8) comes from the fact that the household spends all domestic currency on domestic goods and on acquiring the vehicle currency. The result (4.9) comes from the constraint $f_{i1}^{1j} = s_{1j}^a m_{ij}' = s_{1j}^a n_j p_j c_{ij}$ for $j \neq i, 1$. The result (4.10) comes from (4.4).

(4) The derivation for (4.13). For country 1, it is straightforward to verify that optimal consumption satisfies:

$$s_{1i}^a p_i c_{1i} = p_1 c_{11}, \text{ for } i \neq 1. \tag{A.7}$$

As a vehicle currency, currency 1 will be held by residents of all countries. This means that, compared to the STE, it is no longer true that $m_{11} = 1/n_1$. In fact, since $n_1 m_{11} + \sum_{i \neq 1} n_i m_{i1} = 1$, using (4.10), it must be the case that normalized holdings of currency 1 by country 1 residents are given by (4.13).

(5) The proof of Proposition 4.1. The proof amounts to deriving (4.17), (4.18) and (4.19). For (4.17), substitute f and p_1 from the (4.8), (4.9), (4.10), and (4.14) into (4.16). We get:

$$\left(n_i - \frac{\gamma_1 \phi_1}{n_1} \right) (1 - n_1 m_{11}) - \left[n_i - \frac{\phi_1}{n_1} + \frac{\beta n_i}{\gamma_1} \sum_{j \neq 1} \frac{s_{1j}^b}{\delta_j} \right]$$
$$= \left(\frac{1}{\gamma_1} - 1 \right) \beta (1 - n_i - n_1) \frac{s_{1i}^b}{\delta_i} - \left[n_1 + \frac{\beta}{\gamma_1} (1 - n_1) \right] \frac{s_{1i}^b}{\delta_i}.$$

Summing over $i \neq 1$ and using (4.13) reversely, we have:

$$\sum_{i \neq 1} \frac{s_{1i}^b}{\delta_i} = (1 - n_1 m_{11}) \left[1 - \frac{\gamma_1}{n_1} \left(1 - \frac{(N-1)\phi_1}{n_1} \right) \right] + \frac{1}{n_1} \left(1 - \frac{(N-1)\phi_1}{n_1} \right) - 1.$$

Substituting this result into the left-hand side of the previous equation yields (4.17), where

$$D_i \equiv \frac{(1 - \frac{\beta}{\gamma_1}) n_i - \frac{\phi}{n_1} + \frac{\beta n_i}{\gamma_1 n_1} \left(1 - \frac{(N-1)\phi}{n_1} \right)}{\delta_i - n_i + (\beta/\gamma_1) n_i}, \tag{A.8}$$

$$E_i \equiv \frac{(1 - \frac{\beta}{\gamma_1})) n_i - \frac{\gamma_1 \phi}{n_1} + \frac{\beta n_i}{n_1} \left(1 - \frac{(N-1)\phi}{n_1} \right)}{\delta_i - n_i + (\beta/\gamma_1) n_i}. \tag{A.9}$$

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Substituting (4.17) into (4.13) yields (4.19).

For (4.18), use (4.16) to rewrite (4.15) as follows:

$$s_{1i}^a = \frac{n_i f_{ii}^{1i} s_{1i}^b + \phi p_1}{n_i f_{ii}^{1i} - \phi p_i}.$$

Substituting f_{ii}^{1i} from (4.8) and s_{1i}^b from (4.17) yields (4.18).

(6) The derivation in Case A. We derive (4.20) and (4.21). Since countries are of equal size in Case A, S_{1i}^{bVCE} and S_{1i}^{aVCE} are independent of i. Then we can write (4.15) as:

$$s^{bVCE}\frac{(N-1)}{N} = \frac{m_{11}}{N^2} + \frac{(1-m_{11}/N)}{N-1} - p_1\phi$$

$$= \frac{1}{N}\left(1 - s^{bVCE}\frac{(N-2)(N-1)}{N}\right) + s^{bVCE}\frac{(N-2)}{N} - N\phi\left(1 - s^{bVCE}\frac{(N-2)(N-1)}{N}\right) \quad \text{(A.10)}$$

The first line is explained as follows. The supply of peripheral currency i to the $1i$ trading post originates with the demand of country i households for non-i goods, which equals their money holdings $n_i m_{ii}$ $(= 1)$ times the measure of non-i goods, which is $1 - n_i = (N-1)/N$. Country i sellers then receive the bid price s_{1i}^{bVCE} per unit of currency. The demand for currency i comes from residents of country 1 and country $j \notin \{1, i\}$. First, country 1 residents' total nominal demand for goods is $n_1 m_{11} = m_{11}/N$, and thus their demand for country i goods is $n_i n_1 m_{11} = m_{11}/N^2$. Second, residents of each peripheral country $j \notin \{1, i\}$ exchange currency 1 for currency i. In total, the amount of currency 1 held by peripheral countries is equal to $1 - m_{11}/N$, so the amount per country is $(1 - m_{11}/N)/(N-1)$. An amount $1/(N-2)$ of this is spent on currency i, but there are $N-2$ such countries. Hence, $(1-m_{11}/N)/(N-1)$ represents the total spending on currency i coming from peripheral countries. However, the supply of currency 1 to the $1i$ market is reduced by the amount $p_1\phi$, which is the amount of currency 1 that needs to be held by the $1i$ trading post manager, to cover the fixed cost of setting up the post.

The second line of (A.10) comes from expanding the definitions of m_{11} and p_1 from (4.13) and (4.14). Note that there is a simultaneity here in that both the supply *and* demand for peripheral currency depends on the equilibrium bid price under VCE. Intuitively, the equilibrium bid price determines how much of currency 1 can be taken on to the next trading post.

After re-arranging (A.10), we obtain the solution for s^{bVCE} in Case A as (4.20). Then, (4.21) comes from (4.18).

(7) The derivation in Case C. Substituting the hypotheses in Case C, $\{\gamma_1 = 1, \beta \to 1, n_1 = n, n_i = (1-n)/(N-1), i > 1\}$, we can use (4.17) and (4.18) to obtain s_{1i}^{aVCE} and s_{1i}^{bVCE} in this case. Accordingly, (4.5), (4.6), (4.7) and (4.11) together yield:

$$c_{1i}^{VCE} = \Omega_C(n, N), \quad \text{(A.11)}$$

$$c_{ij}^{VCE} = \left[1 - \frac{\phi(N-1)}{n(1-n)}\right]\Omega_C(n, N). \quad \text{(A.12)}$$

where

$$\Omega_C(n, N) = \left[1 - \phi\frac{(N-1)^2}{(1-n)(n+N-2)}\right] \bigg/ \left[1 - \phi\frac{(N-1)(N-2)}{n(n+N-2)}\right].$$

From (A.11) and (3.17), we can show that:

$$c_{1j}^{VCE} - c_{1j}^{STE} = \frac{\phi\rho}{1 - \phi\rho}\left[2 - \phi\frac{(N-1)}{(1-n)n}\right],$$

where $\rho \equiv \frac{(N-2)(N-1)}{n(n+N-2)} < 1$. Under the feasibility condition $\phi\frac{(N-1)}{(1-n)n} < 1$, the above difference is always positive. Moreover, using (A.11) and (3.17), we can verify (4.28) and (4.29).

(8) The derivation in Case D. With the hypotheses in Case D, $\{\beta \to 1, n = n_i = 1/N\}$, the consumption levels of country 1 in a VCE are:

$$c_{11}^{VCE} = \frac{N[1 + (\gamma_1 - 1)(N-1)(1 - \phi(N-2))]}{N(2\gamma_1 - 1) + 2(1 - \gamma_1)}, \tag{A.13}$$

$$c_{1j}^{VCE} = [1 + (\gamma_1 - 1)(N-1)(1 - \phi(N-2))]\Omega_D(\gamma_1). \tag{A.14}$$

where

$$\Omega_D(\gamma_1) = \frac{1 - \phi\frac{N^2}{N-1}}{\gamma_1 - \phi\frac{N(N-2)}{N-1}(\gamma_1(N-1) + 1)}.$$

Analogously, we can derive the consumption for peripheral countries under VCE as:

$$c_{ii} = 1, \tag{A.15}$$

$$c_{i1} = \frac{(1 - \phi N^2)\gamma_1 N}{N(2\gamma_1 - 1) + 2(1 - \gamma_1)}, \tag{A.16}$$

$$c_{ij} = (1 - \phi N^2)\Omega_D(\gamma_1). \tag{A.17}$$

(9) Proof of Proposition 4.2:

Let $n = 1/N$. Derive s^b and s^a from (4.17) and (4.18), where D_i and E_i can be simplified as

$$D = \frac{1 + \frac{\beta}{\gamma_1}(N-1)}{1 + \beta(N - 1 + \frac{1}{\gamma_1})}(1 - N^2\phi_1)$$

$$E = \frac{(1 - \gamma_1)\left(1 - \frac{\beta}{\gamma_1}\right) + [\beta(N-1) + \gamma_1](1 - N^2\phi_1)}{1 + \beta(N - 1 + \frac{1}{\gamma_1})}$$

Using these, the solutions for s_{1i}^b and s_{1i}^a/s_{1i}^a may be written

$$\frac{s_{1i}^b}{\beta + (1-\beta)2/N} = \frac{N(1 - N^2\phi)}{\beta(N-1)(N-2)(1 - N^2\phi) + N - \beta(\frac{1}{\gamma_1} - 1)(N-2)}$$

$$\frac{s_{1i}^a}{s_{1i}^b} = \frac{\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi}{(1-N^2\phi)\left[\beta(N-2)+1-(\beta(N-2)+2)N\phi\right]} \tag{A.18}$$

The solutions for m_{11} and p_1 are

$$1 - n_1 m_{11} = \frac{1-N^2\phi_1}{\gamma_1(1-N^2\phi_1)+\frac{1}{N-1}\left[\frac{\gamma_1 N}{\beta(N-2)}-(1-\gamma_1)\right]}$$

$$p_1 = \frac{N\left[N-\beta(\frac{1}{\gamma_1}-1)(N-2)\right]}{\beta(N-1)(N-2)(1-N^2\phi_1)+N-\beta(\frac{1}{\gamma_1}-1)(N-2)}$$

Then, using these solutions in the VCE formulas for consumption (4.5), (4.6), and (4.7), we get, for $i \neq 1$ and $j \neq i, 1$, we have:

$$c_{ii} = \frac{1}{\beta+(1-\beta)2/N} \tag{A.19}$$

$$c_{i1} = N\frac{1-N^2\phi_1}{N-\beta\left(\frac{1}{\gamma_1}-1\right)(N-2)} \tag{A.20}$$

$$c_{ij} = \frac{\beta\left(1-N^2\phi_1\right)\left[\beta(N-2)+1-(\beta(N-2)+2)N\phi_j\right]}{\gamma_1\left[\beta+(1-\beta)2/N\right]\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi_1\right]}. \tag{A.21}$$

For country 1, consumption is:

$$c_{11} = N\frac{\beta\left(1-\frac{1}{\gamma_1}\right)(N-2)\left[1-(N-1)N\phi_1\right]+1}{N-\beta(\frac{1}{\gamma_1}-1)(N-2)} \tag{A.22}$$

$$c_{1i} = N\frac{\left[1+\beta(1-\frac{1}{\gamma_1})(N-2)(1-(N-1)N\phi_1)\right]}{\left[\beta(N-2)+1-\beta(N-1+\frac{1}{\gamma_1})(N-2)N\phi_1\right]} \times \frac{[\beta(N-2)+1-(\beta(N-2)+2)N\phi_i]}{\beta(N-2)+2} \tag{A.23}$$

Then, using (A.18) through (A.23), parts (i)-(v) of the proposition can be verified.

B. Derivations for Section 6.1

The optimal choices of a country 2 household yield:

$$p_2 c_{22} = \frac{1}{s_{12}^b}p_1 c_{21} = s_{23}^a p_3 c_{23} = \frac{\gamma_{1(+1)}s_{1j(+1)}^a}{\beta s_{12}^b}p_{j(+1)}c_{2j(+1)}, \ j \notin I.$$

As before, $m_{22} = 1/n_2$, $m_{j2} = 0$ $(j \neq 1, 2)$, and $p_2 = 1/n_2$. Adding up country 2's spending of currency 2, invoking stationarity, and substituting the first-order conditions for c yields:

$$c_{22} = \frac{1}{\delta_2^d},$$

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where $\delta_2^d = n_2 + n_1 + n_3 + \beta(1 - n_1 - n_2 - n_3)$. The household's consumption levels of other goods can be calculated accordingly. Also, for $j \notin I$, the household's optimal decisions on the quantities of currency trade yield:

$$f_{22}^{12} = \frac{1}{n_2} - p_2 n_2 c_{22} - n_3 s_{23}^a p_3 c_{23} = \frac{n_1 + \beta(1 - n_1 - n_2)}{n_2 \delta_2^d}, \tag{B.1}$$

$$f_{22}^{23} = \frac{n_3}{n_2 \delta_2^d}, \quad f_{21}^{1j} = n_j s_{1j}^a p_j c_{2j} = \frac{\beta}{\gamma_1} \frac{s_{12}^b n_j}{n_2 \delta_2^d}$$

$$m_{21} = \sum_{j \notin I} f_{21}^{1j} = \frac{\beta}{\gamma_1} \frac{(1 - n_1 - n_2 - n_3) s_{12}^b}{n_2 \delta_2^d}. \tag{B.2}$$

To compute exchange rates at the 12 post and the 13 post after the deviation by countries 2 and 3, we count the total amount of currency 1 that is held by the peripheral countries at the beginning of a period as follows:

$$1 - n_1 m_{11} = n_2 m_{21} + n_3 m_{31} + \sum_{j \notin I} n_j m_{j1}.$$

At the 12 post, bid/ask prices satisfy the following conditions:

$$s_{12}^a \left(f_{22}^{12} - \frac{\phi}{n_2} \right) = f_{11}^{12} + \sum_{j \notin I} f_{j1}^{12} \tag{B.3}$$

$$s_{12}^b f_{22}^{12} = f_{11}^{12} + \sum_{j \notin I} f_{j1}^{12} - p_1 \phi. \tag{B.4}$$

At the 13 post, the conditions are analogous. At the $1j$ post ($j \notin I$), the conditions are:

$$s_{1j}^a \left(f_{jj}^{1j} - \frac{\phi}{n_j} \right) = f_{11}^{1j} + f_{21}^{1j} + f_{31}^{1j} + \sum_{j \notin I \cup \{j\}} f_{i1}^{1j} \tag{B.5}$$

$$s_{1j}^b f_{jj}^{1j} = f_{11}^{1j} + f_{21}^{1j} + f_{31}^{1j} + \sum_{j \notin I \cup \{j\}} f_{i1}^{1j} - p_1 \phi. \tag{B.6}$$

These equations determine the exchange rate at each post involving currency 1.

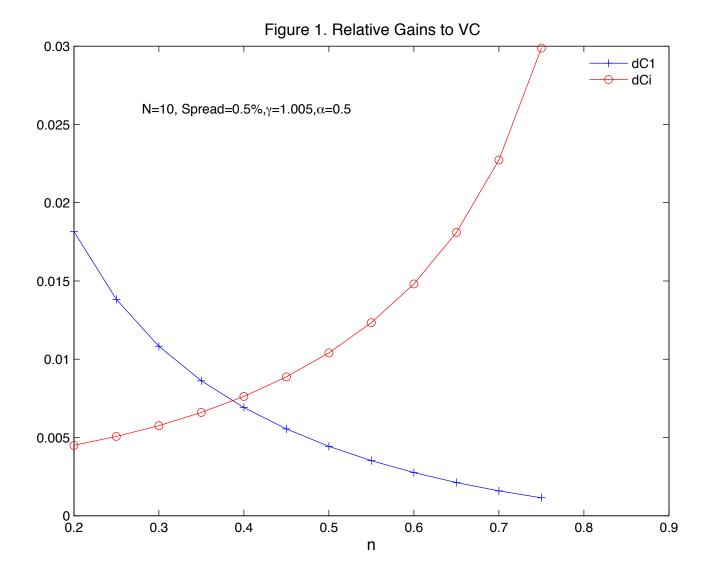## C. Additional Empirical Evidence on the Role of the US Dollar and Foreign Exchange Markets

The paper is based partly on the empirical observation of the importance of the US dollar in world trade. Table A1 provides some evidence on this. It shows, for various years, that the US dollar had a large share in both export, and especially import invoicing for Japan, Korea, Australia and the UK. Other evidence is presented in Devereux et al. (2010).

Figure A1 provides evidence that currencies with larger bilateral trading volume had on average lower foreign exchange spreads in foreign exchange interbank markets. This data is taken from http://www.mt4i.com/spread/broker.aspx?brokerid=3.

Table A1. US dollar use in invoicing imports and exports for selected countries (%)[c]

Country	Observation year	US $ share in exporting invoicing	US $ share in importing invoicing
US	1992-1996	98.0	92.8
	1995	92.0	80.7
	2003	99.8	92.8
Japan	1995	52.7	
	2001	52.4	70.7
Korea	1995	88.1	
	2001	84.9	82.2
Australia	2002	67.9	50.1
	2006	75.3	51.4
	2007	74.3	52.0
United Kingdom	1999	27.0	30.0
	2002	26.0	37.0

[c] Source: Devereux, Shi and Xu (2010).

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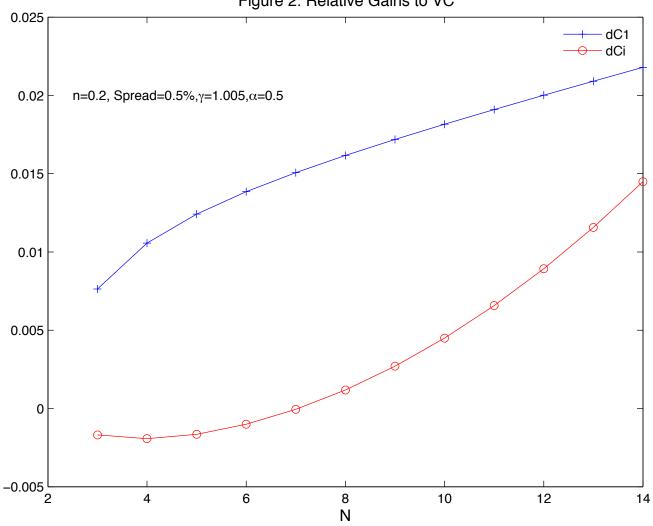Figure 1. Relative Gains to VC

N=10, Spread=0.5%, γ=1.005, α=0.5

Figure 2. Relative Gains to VC

Figure 3. Trade−off Between Inflation and Size
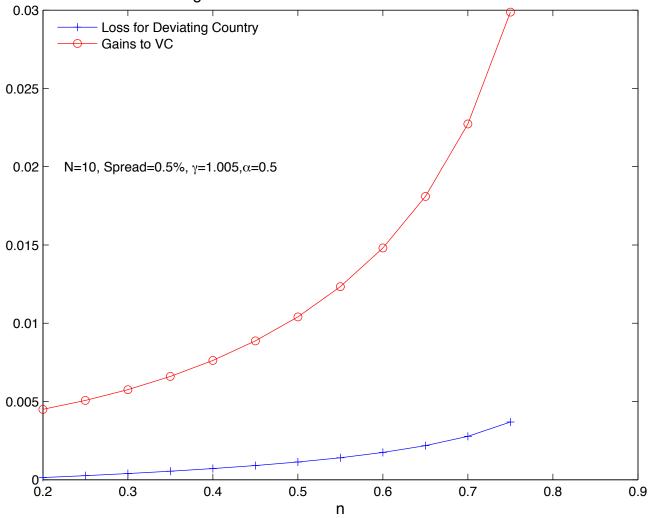
Figure 4. Relative Gains to VC

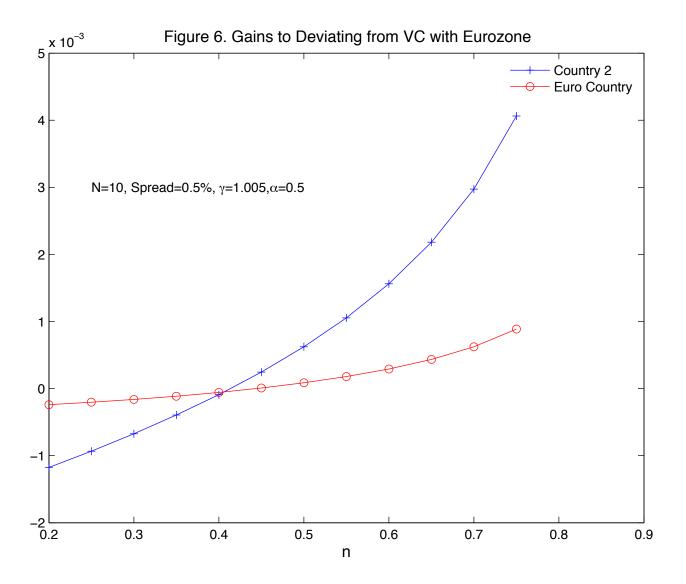N=10, Spread=0.05%, $\gamma=1$, $\alpha=0.5$

Figure 5. Relative Gains to Deviation from VC

Loss for Deviating Country
Gains to VC

N=10, Spread=0.5%, γ=1.005, α=0.5

n

Figure 6. Gains to Deviating from VC with Eurozone

N=10, Spread=0.5%, γ=1.005, α=0.5

Figure A1